AIEEE 2005
MATHEMATICS

1. The differential equation representing the family of curves $y^2 = 2c\left(x + \sqrt{C}\right)$, where $C > 0$, is a parameter, is of order and degree as follows :

(1) order 1, degree 3 (2) order 2, degree 2 (3) order 1, degree 2 (4) order 1, degree 1

Ans. (1)

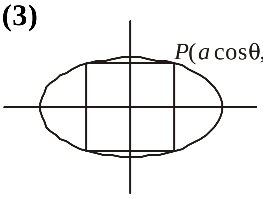
Sol.: The given equation is differentiated once and c is eliminated which gives $y^2 + 4x^2y'^2 - 4xyy' = 4yy'^{3.}$ Hence the order is 1 and degree is 3.

2. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(1) $\sqrt{ab}$ (2) $\frac{a}{b}$ (3) $2ab$ (4) ab

Ans. (3)

Sol.:



From the figure, it is clear that area of the rectangle = $4ab \cos\theta \sin\theta$

$= 2ab \sin 2\theta$

Hence, maximum area = $2ab$.

3. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \cdots + \frac{1}{n}\sec^2 1\right]$ equals

(1) tan 1 (2) $\frac{1}{2}\tan 1$ (3) $\frac{1}{2}\sec 1$ (4) $\frac{1}{2}\operatorname{cosec} 1$

Ans. (2)

Sol.: The general term $T_r = \frac{r}{n^2}\sec^2\left(\frac{r}{n}\right)^2$. Put $\frac{r}{n} = x$ and $\frac{1}{n} = dx$ which gives the value of limit

$$\int_0^1 x \sec^2 x^2\, dx = \frac{\tan 1}{2}$$

4. If the cube root of unity are 1, w, w^2 then roots of equation $(x-1)^3 + 8 = 0$, are

(1) $-1, 1-2w, 1-2w^2$ (2) $-1, 1+2w, 1+2w^2$

(3) $-1, -1+2w, -1-2w^2$ (4) $-1, -1, -1$

Ans. (1)

Sol.: The given equation is $(x-1)^3 + 8 = 0$

This implies $\left(\frac{x-1}{2}\right)^3 = -1 \Rightarrow \left(\frac{x-1}{2}\right) = -1, -\omega$ and $-\omega^2$

$\Rightarrow \quad x = -1, 1 - 2\omega$ and $1 - 2\omega^2$

5. If $A^2 - A + I = 0$, then the inverse of A is

(1) $A - I$ (2) $I - A$ (3) $A + I$ (4) A

Ans. (2)

Sol.: Multiplying the given equation by A^{-1}, we get $A - I + A^{-1} = 0 \Rightarrow A^{-1} = I - A$.

6. Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is

(1) an equivalence relation (2) reflexive and symmetric only

(3) reflexive and transitive only (4) reflexive only

Ans. (3)

Sol.: Since (3, 3), (6, 6), (9, 9), (12, 12) are the members of $R \Rightarrow R$ is reflexive.

Again, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \Rightarrow R$ is transitive.

7. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately

(1) 25.5 (2) 24.0 (3) 22.0 (4) 20.5

Ans. (3)

Sol.: $\text{mode} = \frac{\text{mean} + 2\text{median}}{3} = \frac{21 + 2 \times 22}{3} = 21.66 \simeq 22$.

8. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is

(1) $x^2 + 4y + 2 = 0$ (2) $x^2 - 4y + 2 = 0$ (3) $y^2 - 4x + 2 = 0$ (4) $y^2 + 4x + 2 = 0$

Ans. (3)

Sol.: Let the point Q be $(2t^2, 4t)$ and mid point of PQ be (h, k) then $h = \frac{1 + 2t^2}{2}$ and $k = \frac{0 + 4t}{2}$

Eliminating t, we get $y^2 - 4x + 2 = 0$.

9. If C is the mid point of AB and P is any point outside AB, then

(1) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$ (2) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$ (3) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$ (4) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$

Ans. (3)

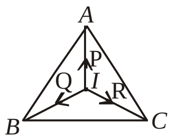
Sol.: The position vector of mid point of AB is $\overrightarrow{PC} = \frac{\overrightarrow{PA} + \overrightarrow{PB}}{2}$

10. *ABC* is a triangle. Forces **P**, **Q**, **R** acting along *IA*, *IB* and *IC* respectively are in equilibrium, where *I* is the incentre of ΔABC. Then **P** : **Q** : **R** is

(1) $\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$ (2) $\cos A : \cos B : \cos C$

(3) $\sin A : \sin B : \sin C$ (4) $\sin\frac{A}{2}:\sin\frac{B}{2}:\sin\frac{C}{2}$

Ans. (1)

Sol.: 

$\angle BIC = \pi - \left(\frac{B+C}{2}\right)$, $\angle AIB = \pi - \left(\frac{A+B}{2}\right)$, $\angle AIC = \pi - \left(\frac{A+C}{2}\right)$

Applying Lami's theorem, we get, $\frac{P}{\sin\frac{B+C}{2}} = \frac{Q}{\sin\frac{A+C}{2}} = \frac{R}{\sin\frac{A+B}{2}}$

$\Rightarrow \quad \frac{P}{\cos\frac{A}{2}} = \frac{Q}{\cos\frac{B}{2}} = \frac{C}{\cos\frac{C}{2}}$

11. In a triangle *PQR*, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then

(1) $b = c$ (2) $b = a + c$ (3) $a = b + c$ (4) $c = a + b$

Ans. (4)

Sol.: $\tan\frac{P}{2} + \tan\frac{Q}{2} = -\frac{b}{a}$ and $\tan\frac{P}{2}\tan\frac{Q}{2} = \frac{C}{a}$, $\tan\left(\frac{P+Q}{2}\right) = 1$ gives $a + b = c$.

12. If the coefficient of *r*th, $(r + 1)$th and $(r + 1)$th and $(r + 2)$th terms in the binomial expansion of $(1 + y)^m$ are in A.P., then *m* and *r* satisfy the equation

(1) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$ (2) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

(3) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$ (4) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$

Ans. (1)

Sol.: Since ${}^mC_{r-1}, {}^mC_r, {}^mC_{r+1}$ are in A.P. $\Rightarrow$ $2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$

which gives $m^2 - m(4r+1) + 4r^2 - 2 = 0$

13. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is interval

(1) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (3) $\left(0, \frac{\pi}{2}\right)$ (4) $\left[0, \frac{\pi}{2}\right)$

Ans. (2)

Sol.: Since $\tan^{-1}\frac{2x}{1-x^2}$ is an increasing function so the range B will be $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

14. If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation

(1) $\frac{a}{b} = 1$ (2) $ab = 1$ (3) $a - b = 1$ (4) $a + b = 1$

Ans. (1)

Sol.: The coefficient of x^7 in the expansion of $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ = ${}^{11}C_5\, a^6\, b^{-5}$ and coefficient of x^{-7} in the expansion of $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$ = ${}^{11}C_6\, a^5\, b^{-6}$. On equating, we get $a/b = 1$.

15. If $w = \frac{z}{z - \frac{1}{3}i}$ and $|w| = 1$, then z lies on

(1) a straight line (2) a parabola (3) an ellipse (4) a circle

Ans. (1)

Sol.: $\omega = \frac{z}{z - \frac{1}{3}i} \Rightarrow \frac{|z|}{\left|z - \frac{i}{3}\right|} = 1 \Rightarrow |z| = \left|z - \frac{i}{3}\right|$ which is bisector of the line joining origin and the point (i/3).

16. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$, then $f(x)$ is a polynomial of degree

(1) 3 (2) 2 (3) 1 (4) 0

Ans. (2)

Sol.: $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$, Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 1 & x+b^2x & x+c^2x \\ 1 & 1+b^2x & x+c^2x \\ 1 & x+b^2x & 1+c^2x \end{vmatrix} = (1-x)^2$$

17. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg z_1 – arg z_2 is equal to

(1) 0 (2) $\frac{-\pi}{2}$ (3) $\frac{\pi}{2}$ (4) $-\pi$

Ans. (1)

Sol.: Given that $|z_1 + z_2| = |z_1| + |z_2|$ which is possible when Arg z_1 = Arg z_2 $\Rightarrow$ Arg z_1 – Arg z_2 = 0.

18. The value of a for which the sum of the squares of the roots of the equation

$x^2 - (a-2)x - a - 1 = 0$ assume the least value is

(1) 3 (2) 2 (3) 1 (4) 0

Ans. (3)

Sol.: Let α and β be roots of equation, then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a-1)^2 + 5$ which gives $a = 1$.

19. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals

(1) 2 (2) 1 (3) –2 (4) 3

Ans. (2)

Sol.: The difference of roots = 1 $\Rightarrow$ $b^2 - 4c = 1$.

20. The system of equations

$\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

has no solution, if α is

(1) not –2 (2) 1 (3) –2 (4) either –2 or 1

Ans. (3)

Sol.: The value of the coefficient determinent $D = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$ which gives $(\alpha - 1)^2 [\alpha + 2] = 0$. So,

$\alpha = 1$ or $\alpha = -2$ but for $\alpha = 1$ there are infinite number of solutions of the given system.

21. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(1) ${}^{56}C_3$ (2) ${}^{56}C_4$ (3) ${}^{55}C_4$ (4) ${}^{55}C_3$

Ans. (2)

Sol.: ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3 = {}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3.$

$= {}^{50}C_3 + {}^{50}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 = {}^{56}C_4.$

22. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction

(1) $A^n = nA + (n-1)I$ (2) $A^n = 2^{n-1}A + (n-1)I$

(3) $A^n = nA - (n-1)I$ (4) $A^n = 2^{n-1}A - (n-1)I$

Ans. (3)

Sol.: $A = I + \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$. The two matrices on the right commute, hence by the Binomial theorem

$A^n = I + n\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$ because powers higher than or equal to 2 of the matrix $\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$ are 0.

23. If the letters of word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(1) 603 (2) 602 (3) 601 (4) 600

Ans. (3)

Sol.: The alphabatical order of letters is A, C, H, I, N, S. The number of words which begin with any of the five letters A, C, H, I, N are 120 × 5 = 600. The next word will be SACHIN so its rank. is 601.

24. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$ then x, y, z are in

(1) Arithmetic – Geometric Progression (2) HP

(3) GP (4) AP

Ans. (2)

Sol.: $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n \Rightarrow x = \frac{1}{1-a}$, $y = \frac{1}{1-b}$ and $z = \frac{1}{1-c}$

Since a, b, c are in A.P. x, y, z are in A.P.

25. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{\frac{3}{2}} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as

(1) $-\frac{3}{8}x^2$ (2) $\frac{x}{2}-\frac{3}{8}x^2$ (3) $1-\frac{3}{8}x^2$ (4) $3x+\frac{3}{8}x^2$

Ans. (1)

Sol.: Upto terms of order x^2

$$\frac{(1+x)^{\frac{3}{2}} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}} = \left[1+\frac{3}{2}x+\frac{3}{8}x^2 - 1 - \frac{3}{2}x - \frac{3}{4}x^2\right]\left[1+\frac{1}{2}x+\frac{3}{8}x^2\right] = -\frac{3}{8}x^2$$

26. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(1) $4\sin^2\alpha$ (2) $-4\sin^2\alpha$ (3) $2\sin2\alpha$ (4) 4

Ans. (1)

Sol.: Given that $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, $\Rightarrow$ $\cos(\cos^{-1}x - \cos^{-1}y/2) = \cos\alpha$

$\Rightarrow$ $\frac{xy}{2} + \sqrt{1-x^2}\sqrt{1-y^2/4} = \cos\alpha$ $\Rightarrow$ $4x^2 + y^2 - 4xy\cos\alpha = 4\sin^2\alpha$

27. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P., then $\sin A$, $\sin B$, $\sin C$ are in

(1) Arithmetic – Geometric Progression (2) H.P.

(3) G.P. (4) A.P.

Ans. (4)

Sol.: The altitudes of the triangle are $\frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$ are given in H.P.

$\Rightarrow$ $\sin A$, $\sin B$, $\sin C$ are in A.P.

28. In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then $2(r + R)$ equals

(1) $a + b + c$ (2) $c + a$ (3) $b + c$ (4) $a + b$

Ans. (4)

Sol.: For a right angled triangle $R = c/2$ and $r = \frac{ab}{a+b+c}$ $\Rightarrow$ $2(r+R) = (a+b)$

29. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

(1) $\left(-\infty, \frac{1}{3}\right]$ (2) $(-\infty, -4]$ (3) $(-\infty, \infty)$ (4) $[2, \infty)$

Ans. (1)

Sol.: Given that $f(x) = 3x^2 - 2x + 1 \Rightarrow f'(x) = 6x - 2 \geq 0 \Rightarrow x \geq \frac{1}{3}$

30. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\underset{x\to\alpha}{Lim}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(1) $\frac{-a^2}{2}(\alpha-\beta)^2$ (2) $\frac{1}{2}(\alpha-\beta)^2$ (3) $\frac{a^2}{2}(\alpha-\beta)^2$ (4) 0

Ans. (3)

Sol.: $\underset{x\to\alpha}{Lim}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2} = \lim_{x\to\alpha}\frac{1-\cos a(x-\alpha)(x-\beta)}{(x-\alpha)^2} = \frac{a^2}{2}(\alpha-\beta)^2$ because $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$.

31. The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta + \theta\cos\theta)$ at any point 'θ' is such that

(1) it passes through $\left(a\frac{\pi}{2}, -a\right)$ (2) it is at a constant distance from the origin

(3) it passes through the origin (4) it makes angle $\frac{\pi}{2}+\theta$ with the *x*-axis

Ans. (2)

Sol.: The equation of normal to the curve at any point is $y \sin\theta + x\cos\theta = a$ which is at a constant distance from origin.

32. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then

(1) $f(6) < 5$ (2) $f(6) = 5$ (3) $f(6) \geq 8$ (4) $f(6) < 8$

Ans. (3)

Sol.: Applying Lagrange's mean value theorem, $\frac{f(6)-f(1)}{6-1} = f'(c) \ \forall c \in (1,6)$

$\Rightarrow \frac{f(6)+2}{5} \geq 2 \Rightarrow f(6) \geq 8$

33. If f is a real-valued differentiable function satisfying $|f(x) - f(y)| \le (x - y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals

(1) 2 (2) 1 (3) –1 (4) 0

Ans. (4)

Sol.: $|f(x) - f(y)| \le (x - y)^2$, $x, y \in R$ taking the limit as $x \to y$, we get $|f'(x)| \le 0 \Rightarrow f'(x) = 0 \Rightarrow f(x)$ is a constant function $\Rightarrow f(1) = 0$

34. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h) = 5$, then $f'(1)$ equals

(1) 5 (2) 6 (3) 3 (4) 4

Ans. (1)

Sol.: Since $\lim_{h\to 0}\frac{f(1+h) - f(1)}{h}$ exists and $\lim_{h\to 0}\frac{1}{h}f(1+h)$ exists, it follows that $\lim_{h\to 0}\frac{f(1)}{h}$ exists

Hence, $f(1) = 0$ and $f'(1) = \lim_{h\to 0}\frac{f(1+h) - f(1)}{h} = \lim_{h\to 0}\frac{f(1+h)}{h} = 5$.

35. $\int\left\{\frac{(\log x - 1)}{1 + (\log x)^2}\right\}^2 dx$ is equal to

(1) $\frac{xe^x}{1+x^2} + C$ (2) $\frac{x}{(\log x)^2 + 1} + C$ (3) $\frac{\log x}{(\log x)^2 + 1} + C$ (4) $\frac{x}{x^2 + 1} + C$

Ans. (2)

Sol.: Let $I = \int\left\{\frac{(\log x - 1)}{1 + (\log x)^2}\right\}^2 dx$. Put $\log x = t$, then $I = \int e^t\left(\frac{1}{(1+t^2)} - \frac{2t}{(1+t^2)^2}\right)dt$

$$= \frac{e^t}{1+t^2} + C = \frac{x}{1 + (\log x)^2} + C$$

36. A spherical ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

(1) $\frac{1}{54\pi}$ cm/min. (2) $\frac{5}{6\pi}$ cm/min. (3) $\frac{5}{36\pi}$ cm/min. (4) $\frac{1}{18\pi}$ cm/min.

Ans. (4)

Sol.: Let $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt} = 4\pi r^2\frac{dr}{dt} \Rightarrow 50 = 4\pi(225)\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{18\pi}$ cm/m

37. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x=\frac{\pi}{4}$ and $x=\beta>\frac{\pi}{4}$ is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\,\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is

(1) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$ (2) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$ (3) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (4) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$

Ans. (2)

Sol.: Let $A=\int_{\pi/4}^{\beta} f(x)\,dx=\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\,\beta$. Hence $\frac{dA}{d\beta}=f(\beta)=\sin\beta+\beta\cos\beta-\frac{\pi}{4}\sin\beta+\sqrt{2}$

$\Rightarrow f\left(\frac{\pi}{2}\right)=1-\frac{\pi}{4}+\sqrt{2}$

38. Let $f: R \to R$ be a differentiable function having $f(2) = 6$, $f'(2)=\left(\frac{1}{48}\right)$. Then $\lim_{x\to 2}\int_{6}^{f(x)}\frac{4t^3}{x-2}dt$ equals

(1) 12 (2) 18 (3) 24 (4) 36

Ans. (2)

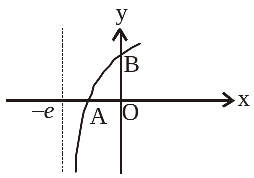
Sol.: By L'Hospital's rule $\lim_{x\to 2}\int_{6}^{f(x)}\frac{4t^3}{x-2}dt=\lim_{x\to 2} f'(x)\,4[f(x)]^3=\frac{4}{48}\times 216=18$

39. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is

(1) 3 (2) 4 (3) 1 (4) 2

Ans. (3)

Sol.:



Required area = Area of the sector OAB =

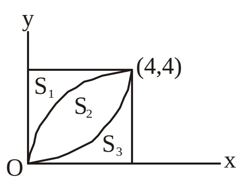
$$\int_{1-e}^{0}\ln(x+e)\,dx=\int_{1}^{e}\ln u\,du=[u\ \ln u\ -u]_1^e=1$$

40. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

(1) 2 : 1 : 2 (2) 1 : 1 : 1 (3) 1 : 2 : 1 (4) 1 : 2 : 3

Ans. (2)

Sol.:



$$S_3=\int_0^4\frac{x^2}{4}dx=\frac{64}{12},\quad S_1=\int_0^4\frac{y^2}{4}dy=\frac{64}{12},\quad S_2=16-S_1-S_3=16-\frac{128}{12}=\frac{64}{12}$$

41. If $I_1 = \int_0^1 2^{x^2}\, dx,\ I_2 = \int_0^1 2^{x^3}\, dx,\ I_3 = \int_1^2 2^{x^2}\, dx,$ and $I_4 = \int_1^2 2^{x^3}\, dx$ then

(1) $I_3 = I_4$ (2) $I_3 > I_4$ (3) $I_2 > I_1$ (4) $I_1 > I_2$

Ans. (4)

Sol.: $2^{x^2} > 2^{x^3}$ if $0 < x < 1$. Hence $I_1 > I_2$.

42. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is

(1) above the x-axis at a distance of $\frac{3}{2}$ from it (2) above the x-axis at a distance of $\frac{2}{3}$ from it

(3) below the x-axis at a distance of $\frac{3}{2}$ from it (4) below the x-axis at a distance of $\frac{2}{3}$ from it

Ans. (3)

Sol.: The equation of the required line can be written as $ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$. The coefficient of x must be zero which implies λ equals $-a/b$. Substituting this value the equation of the line is $y = -3/2$.

43. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is

(1) $\log\left(\frac{y}{x}\right) = cx$ (2) $\log\left(\frac{x}{y}\right) = cy$ (3) $y\log\left(\frac{x}{y}\right) = cy$ (4) $x\log\left(\frac{y}{x}\right) = cy$

Ans. (1)

Sol.: The given equation can be written as $\frac{dy}{dx} = \frac{y}{x}\left(\ln\frac{y}{x}\right) + 1$. Substituting $u = \frac{y}{x}$ gives the differential equation $x\frac{du}{dx} = u\ln u$ whose solution is $\ln u = cx$.

44. If a vertex of a triangle is (1, 1) and the mid points of two sides through the vertex are (–1, 2) and (3, 2), then the centriod of the triangle is

(1) $\left(1, \frac{7}{3}\right)$ (2) $\left(\frac{1}{3}, \frac{7}{3}\right)$ (3) $\left(-1, \frac{7}{3}\right)$ (4) $\left(\frac{-1}{3}, \frac{7}{3}\right)$

Ans. (1)

Sol.: The coordinates of the other two vertices are obtained using section formula and are (–3, 3) and (5, 3). Hence the centroids has coordinates (1, 7/3).

45. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for

(1) infinitely many values of a
(2) exactly two values of a
(3) exactly one value of a
(4) no value of a

Ans. (4)

Sol.: The equatin of the common chord is $5ax + (c - d)y + a + 1 = 0$. This should be the same as $5x + by - a = 0$. Hence $\frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$ which gives the quadratic $a^2 + a + 1 = 0$ which has no real roots.

46. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point. That point is

(1) $(1, -2)$
(2) $\left(1, -\frac{1}{2}\right)$
(3) $(-1, 2)$
(4) $(-1, -2)$

Ans. (1)

Sol.: $\frac{1}{a} + \frac{1}{c} - \frac{2}{b} = 0$. Comparing this with the equation $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ the required point is $(1, -2)$

47. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is

(1) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2\ \ p^2) = 0$
(2) $2ax + 2by - (a^2 + b^2 + p^2) = 0$
(3) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$
(4) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

Ans. (2)

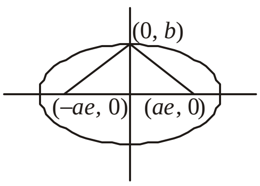
Sol.: Let the centre be (h, k) and suppose the equation of the circle is $x^2 + y^2 - 2hx - 2ky + c = 0$. The orthogonality condition gives $c = p^2$. Substituting this value of c and putting $x = a, y = b$ in the equation of the circle gives the locus $2ax + 2by - (a^2 + b^2 + p^2) = 0$.

48. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

(1) $\frac{1}{4}$
(2) $\frac{1}{\sqrt{3}}$
(3) $\frac{1}{\sqrt{2}}$
(4) $\frac{1}{2}$

Ans. (3)

Sol.:



By Pythagoras theorem $2(b^2 + a^2e^2) = 4a^2e^2$. Hence $b^2 / a^2 = e^2$. For an ellipse $b^2/a^2 = 1 - e^2$ which gives $e = 1/\sqrt{2}$.

49. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is

(1) a hyperbola (2) a parabola (3) an ellipse (4) a circle

Ans. (2)

Sol.: A circle which touches the x axis has equation $x^2 + y^2 + 2gx + 2fy + g^2 = 0$. This touches the given circle if $g^2 + f^2 + 6f + 9 = (f \pm 2)^2$. Hence the locus is parabola.

50. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(1) 45º (2) 30º (3) 0º (4) 90º

Ans. (4)

Sol.: The direction ratios of the two lines are $\left(\frac{1}{2}, \frac{1}{3}, -1\right)$ and $\left(\frac{1}{6}, -1, -\frac{1}{4}\right)$. The dot product of these two vectors is zero. Hence the angle is 90º.

51. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ the value of λ is

(1) $\frac{3}{4}$ (2) $\frac{-4}{3}$ (3) $\frac{5}{3}$ (4) $\frac{-3}{5}$

Ans. (3)

Sol.: The directin ratio of the line is $\mathbf{e} = (1, 2, 2)$. The direction ratio of the normal to the plane is $\left(2, -1, \sqrt{\lambda}\right)$

$\mathbf{n} \cdot \mathbf{e} = 3\sqrt{5+\lambda}\sin\theta$ gives $2\sqrt{\lambda} = \sqrt{5+\lambda}$. So $\lambda = \frac{5}{3}$

52. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

(1) a parabola (2) a hyperbola (3) an ellipse (4) a circle

Ans. (2)

Sol.: Condition for tangency gives $\beta^2 = a^2\alpha^2 - b^2$. Hence the locus is the hyperbola.

53. The distance between the line $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 4\mathbf{k})$ and the plane $\mathbf{r}.(\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = 5$ is

(1) $\frac{3}{10}$ (2) $\frac{10}{3}$ (3) $\frac{10}{9}$ (4) $\frac{10}{3\sqrt{3}}$

Ans. (4)

Sol.: The distance can be obtained by taking any point on the line and any point on the plane and projecting the vector joining these two points along the unit normal to the plane. A point on the line is $2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and the point on the plane is $\mathbf{j}$. The unit normal is $\frac{\mathbf{i} + 5\mathbf{j} + \mathbf{k}}{3\sqrt{3}}$. The required distance is $\frac{10}{3\sqrt{3}}$.

54. For any vector **a**, the value of $(\mathbf{a} \times \mathbf{i})^2 + (\mathbf{a} \times \mathbf{j})^2 + (\mathbf{a} \times \mathbf{k})^2$ is equal to

(1) $2\mathbf{a}^2$ (2) $4\mathbf{a}^2$ (3) $3\mathbf{a}^2$ (4) $\mathbf{a}^2$

Ans. (1)

Sol.: Use $|\mathbf{a}\times\mathbf{i}|^2 = |\mathbf{a}|^2 - (\mathbf{a}\cdot\mathbf{i})^2$ etc. gives $2|\mathbf{a}|^2$.

55. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals

(1) –2 (2) 2 (3) –1 (4) 1

Ans. (1)

Sol.: The centres of the two spheres are (–3, 4, 1) and (5, –2, 1) whose mid point is (1, 1, 1). This lies on the plane. We get $a = -2$.

56. Let a, b and c be distinct non-negative numbers. If the vectors $a\mathbf{i} + a\mathbf{j} + c\mathbf{k}$, $\mathbf{i} + \mathbf{k}$ and $c\mathbf{i} + c\mathbf{j} + b\mathbf{k}$ lie in a plane, then c is

(1) equal to zero (2) the Harmonic Mean of a and b

(3) the Geometric Mean a and b (4) the Arithmetic Mean of a and b

Ans. (3)

Sol.: The scalar triple product of the three vectors must be zero which gives $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

implying $ab = c^2$

57. If **a**, **b**, **c** are non-coplanar vectors and λ is a real number then $[\lambda(\mathbf{a}+\mathbf{b})\ \lambda^2\mathbf{b}\ \lambda\mathbf{c}] = [\mathbf{a}\ \mathbf{b}+\mathbf{c}\ \mathbf{b}]$ for

(1) exactly three values of λ (2) exactly two values of λ

(3) exactly one value of λ (4) no value of λ

Ans. (4)

Sol.: $[\lambda(\mathbf{a}+\mathbf{b})\ \lambda^2\mathbf{b}\ \lambda\mathbf{c}] = \lambda^4[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$, $[\mathbf{a}\ \ \mathbf{b}+\mathbf{c}\ \ \mathbf{b}] = -[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$. Hence the equation has no solution for λ.

58. Let $\mathbf{a} = \mathbf{i} - \mathbf{k}$, $\mathbf{b} = x\mathbf{i} + \mathbf{j} + (1 - x)\mathbf{k}$ and $\mathbf{c} = y\mathbf{i} + x\mathbf{j} + (1 + x - y)\mathbf{k}$. Then [**a**, **b**, **c**] depends on

(1) both x and y (2) neither x nor y (3) only y (4) only x

Ans. (4)

Sol.: $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix} = 1 + 2x$

59. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(1) $\frac{8}{9}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{1}{9}$

Ans. (4)

Sol.: Number of points in the sample space is 27 and the number of points favourable to the event is 3. Hence, the required probability is $\frac{1}{9}$.

60. A random variable X has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals

(1) $1-\frac{3}{e^2}$ (2) $\frac{3}{e^2}$ (3) $\frac{2}{e^2}$ (4) 0

Ans. (1)

Sol.: $P(X > 1.5) = 1 - P(X = 0) - P(X = 1) = 1 - \frac{3}{e^2}$

61. Let A and B be two events such that $P\left(\overline{A \cup B}\right) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P\left(\overline{A}\right) = \frac{1}{4}$, whree $\overline{A}$ stands for component of event A. Then events A and B are

(1) independent but not equally likely (2) mutually exclusive and independent

(3) equally likely and mutually exclusive (4) equally likely but not independent

Ans. (1)

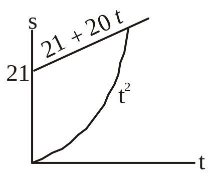
Sol.: $P(A) = \frac{3}{4}, P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{4} \Rightarrow P(B) = \frac{1}{3}$. So, $P(A \cap B) = P(A)P(B)$.

62. A lizard, at initial distance of 21 cm behind an insect, moves from rest with an acceleration of 2 cm/s^2 and pursues the insect which is crawling uniformly along a straight line at a speed of 20 cm/s. Then the lizard catch the insect after

(1) 21 s (2) 24 s (3) 20 s (4) 1 s

Ans. (1)

Sol.:



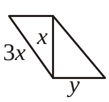
$t^2 = 21 + 20t$ gives $t = 21$ s.

63. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to smaller one is

(1) 3 : 2 (2) $3:2\sqrt{2}$ (3) 2 : 1 (4) $3:\sqrt{2}$

Ans. (2)

Sol.:



By Pythagoras theorem, $9x^2 = x^2 + y^2$. So, $\frac{3x}{y} = \frac{3}{\sqrt{8}}$

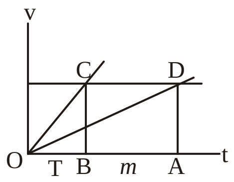
64. Two points A and B move from rest along a straight line with constant acceleration f and f' respectively. If A takes m sec. more than B and describes 'n' units more than B in acquiring the same speed then

(1) $\frac{1}{2}(f + f')m = ff'n^2$ (2) $(f' - f)n = \frac{1}{2} ff'm^2$

(3) $(f - f')m^2 = ff'n$ (4) $(f + f')m^2 = ff'n$

Ans. (2)

Sol.:



Area of triangle OAD – Area of triangle $OCB = n$ gives $n = \frac{1}{2} f'T$

$f(m+T) = f'T \Rightarrow (f' - f)T = fm$. Using the first equation, we get,

$2n(f' - f) = ff'm^2$

65. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with the. The resultant of A and B after combining is displaced through a distance

(1) $\frac{H}{2(A+B)}$ (2) $\frac{H}{A-B}$ (3) $\frac{2H}{A-B}$ (4) $\frac{H}{A+B}$

Ans. (4)

Sol.: When the forces A and B are acting on the plane, the distance of resultant from the force A is equal to $\frac{dB}{A+B}$ where d is the distance between the points of action of the forces A and B. When the couple of moment H is also acting on the plane, then distance of the resultant force from the force $A = \frac{dB}{A+B} - \frac{H}{A+B}$. Thus the position of resultant is shifted by $\frac{H}{A+B}$.

66. The sum of the series : $1 + \frac{1}{4.2!} + \frac{1}{16.4!} + \frac{1}{64.4!} + \dots$ ad inf. is

(1) $\frac{e-1}{2\sqrt{e}}$ (2) $\frac{e+1}{2\sqrt{e}}$ (3) $\frac{e-1}{\sqrt{e}}$ (4) $\frac{e+1}{\sqrt{e}}$

Ans. (2)

Sol.: General term = $\frac{1}{2^n\, n!}$, $n = 0, 2, 4$

$$\sum_{n \text{ even}} \frac{x^n}{n!} = \frac{e^x + e^{-x}}{2}$$

67. If $a_1, a_2, a_3, ..., a_n$ are in G.P., then the determinant $\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to

(1) 4 (2) 2 (3) 1 (4) 0

Ans. (4)

Sol.: $\log a_n, \log a_{n+1}, \log a_{n+2}$ are in A.P.

68. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

(1) $(-\infty, 4)$ (2) $[4, 5]$ (3) $(5, 6]$ (4) $(6, \infty)$

Ans. (1)

Sol.: Discreminent equals $-4(k - 5) \geq 0 \Rightarrow k \leq 5$. The quadratic equation at $x = 5$ must be +ve and sum of the roots must be less than 10. These conditions implie $k^2 - 9k + 20 > 0$. So, $k < 4$.

69. If the equation $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$ has a positive root, which is

(1) greater than or equal to α (2) equal to α

(3) greater than α (4) smaller than α

Ans. (4)

Sol.: The expression $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1$ is the derivative of $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x$. So, by Rolle's theorem, the derivative is 0 at some point between 0 and α.

70. A real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$, where a is a given constant and $f(0) = 1$, $f(2a - x)$ is equal to

(1) $f(a) + f(a - x)$ (2) $f(-x)$ (3) $-f(x)$ (4) $f(x)$

Ans. (3)

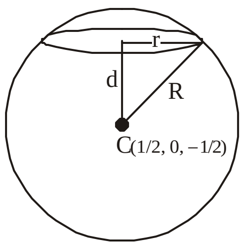
Sol.: Put $y = 0$ in the given functional equation to get $f(a) = 0$. Next put $x = 0$ to get $f(-y) = f(y)$. Next, put $x = y = a$ to get $f(2a) = -1$. Finally, put $x = 2a$ and replace y by x to get $f(2a - x) = -f(x)$.

71. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius

(1) 2 (2) $\sqrt{2}$ (3) 3 (4) 1

Ans. (4)

Sol.:



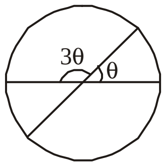
$$r^2 = R^2 - d^2 = \frac{5}{2} - \frac{9}{6} = 1$$

72. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then

(1) $3a^2 + 10ab + 3b^2 = 0$ (2) $3a^2 + 2ab + 3b^2 = 0$

(3) $3a^2 - 10ab + 3b^2 = 0$ (4) $3a^2 - 2ab + 3b^2 = 0$

Ans. (2)

Sol.: 

From the diagram it is clear that θ equals 45º,

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2\left[(a+b)^2 - ab\right]^{1/2}}{a+b} \text{ gives } 3a^2 + 2ab + 3b^2 = 0$$

73. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx,\ a > 0,$ is

(1) $\frac{\pi}{a}$ (2) 2π (3) $a\pi$ (4) $\frac{\pi}{2}$

Ans. (4)

Sol.: $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx = \int_{-\pi}^{\pi} \frac{\cos^2(\pi-\pi-x)}{1+a^{\pi-\pi-x}}\,dx$ gives $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx = \frac{1}{2}\int_{-\pi}^{\pi} \cos^2 x\,dx = \frac{\pi}{2}$.

74. A particle is projected from a point *O* with velocity u at an angle of 60º with the horizontal. When it is moving in a direction at right angles to its direction at *O*, its velocity then is given by

(1) $\frac{2u}{3}$ (2) $\frac{u}{\sqrt{3}}$ (3) $\frac{u}{3}$ (4) $\frac{u}{2}$

Ans. (2)

Sol.: $\mathbf{v} = \mathbf{u} + \mathbf{g}\,t$ $\quad \mathbf{v}\cdot\mathbf{u} = 0$ gives $t = \frac{2|\mathbf{u}|}{\sqrt{3}\,g}$. This implies $|\mathbf{v}| = \frac{|\mathbf{u}|}{\sqrt{3}}$

75. Let $x_1, x_2, ..., x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is

(1) 9 (2) 12 (3) 15 (4) 18

Ans. (4)

Sol.: By Cauchy-Schwarz inequality, $\left|\sum_{i=1}^{n} x_i\right| \le \sqrt{n}\left(\sum x_i^2\right)^{\frac{1}{2}}$ we get $n \ge 16$.