PAPER-1: CHEMISTRY, MATHEMATICS & PHYSICS

Do not open this test booklet until you are asked to do so.

Read carefully the instruction on the Back Cover of this Test Booklet.

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The answer sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 432.
5. There are three parts in the question paper. The distribution of marks subjectwise in each part is as under for each correct response.
   - Part-A - Chemistry (144 marks) - Question No. 1 to 24 consist Four (4) Marks each and Question No. 25 to 30 consist Eight (8) marks each for each correct response.
   - Part-B - Mathematics (144 marks) - Question No. 31 to 32 and 39 to 60 consist Four (4) Marks each and Question No. 33 to 38 consist Eight (8) marks each for each correct response.
   - Part-C - Physics (144 marks) - Question No. 61 to 84 consist Four (4) Marks each and Question No. 85 to 90 consist Eight (8) marks each for each correct response.
6. Candidates will be awarded marks as in instructions No. 5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicated incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the Answer Sheet.
7. Use Blue/Black Ball Point Pen only for writing particulars/marking responses on Side-1 and Side-2 of the answer sheet. Use of pencil is strictly prohibited.
8. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit card inside the examination hall/room.
9. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in 2 pages at the end of the booklet.
10. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
11. The CODE for this booklet is B. Make sure that the CODE printed on Side-2 of the answer sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
12. Do not fold or make any stray marks on the Answer Sheet.

Name of the Candidate (in Capital Letters): ___________________________________________
Roll Number : in figure ____________________________________________________________
     : in words ____________________________________________________________
Examination Centre Number: _______________________________________________________
Name of Examination Centre (in Capital Letters) : ___________________________________
Candidate’s Signature : ___________________  Invigilator Signature ____________________

Amity Institute for Competitive Examinations : Phones: 26963838, 26850005/6, 25573111/2/3/4, 95120-2431839/42
PART-A: CHEMISTRY

1. The IUPAC name of neopentane is:
   (1) 2,2-dimethylpropane  (2) 2-methylpropane
   (3) 2,2-dimethylbutane  (4) 2-methylbutane

   Sol: Ans [1]

\[
\begin{array}{c}
\text{CH}_3 \\
\text{H}_2\text{C} \quad \text{CH}_3 \\
\text{CH}_3
\end{array}
\]

2,2-dimethylpropane

2. Which one of the following reactions of Xenon compounds is not feasible?
   (1) \( 3\text{XeF}_2 + 6\text{H}_2\text{O} \rightarrow 2\text{Xe} + \text{XeO}_3 + 12\text{HF} + 1.5\text{O}_2 \)
   (2) \( 2\text{XeF}_2 + 2\text{H}_2\text{O} \rightarrow 2\text{Xe} + 4\text{HF} + \text{O}_2 \)
   (3) \( \text{XeF}_6 + \text{RbF} \rightarrow \text{Rb}[\text{XeF}_7] \)
   (4) \( \text{XeO}_4 + 6\text{HF} \rightarrow \text{XeF}_6 + 3\text{H}_2\text{O} \)

   Sol: Ans [4]

3. The major product obtained on interaction of phenol with sodium hydroxide and carbon dioxide is:
   (1) Salicylaldehyde  (2) salicylic acid  (3) phthalic acid  (4) benzoic acid

   Sol: Ans [2]

\[
\begin{array}{c}
\text{OH} \\
\text{Ph} + \text{CO}_2 \xrightarrow{(i)\text{NaOH}, 6-7\text{atm}, 125-140\text{C}} \\
\text{PhOH} \xrightarrow{(ii)\text{H}^+} \text{OH} \\
\text{COOH}
\end{array}
\]

Salicylic acid

4. Which of the following statements is incorrect regarding physisorption?
   (1) More easily liquefiable gases are adsorbed readily
   (2) Under high pressure it results into multi molecular layer on adsorbent surface
   (3) Ethalphy of adsorption (\( \Delta H_{\text{adsorption}} \)) is low and positive
   (4) It occurs because of van der Waal’s forces

   Sol: Ans [3]

Adsortion is always exothermic

5. Which of the following has an optical isomer?
   (1) \([\text{CO(en)}(\text{NH}_3)_2]^{2+}\)  (2) \([\text{CO(H}_2\text{O})_4 (\text{en})]^{3+}\)  (3) \([\text{CO(en)}_2(\text{NH}_3)_2]^{1+}\)  (4) \([\text{CO(NH}_3)_3\text{Cl}]^{3+}\)
6. Solid Ba(NO$_3$)$_2$ is gradually dissolved in a 1.0 × 10$^{-4}$M Na$_2$CO$_3$ solution. At what concentration of Ba$^{2+}$ will a precipitate begin to form? (K_{sp} for BaCO$_3$ = 5.1 × 10$^{-9}$)

(1) 5.1 × 10$^{-5}$M
(2) 8.1 × 10$^{-8}$M
(3) 8.1 × 10$^{-7}$M
(4) 4.1 × 10$^{-5}$M

Sol: Ans [1]

K_{sp} = [Ba$^{2+}$][CO$_3^{2-}$]

Na$_2$CO$_3$ $\rightleftharpoons$ 2Na$^+$ + CO$_3^{2-}$

5.1 × 10$^{-9}$ = [Ba$^{2+}$] [1 × 10$^{-4}$]

$\frac{5.1 \times 10^{-9}}{1 \times 10^{-4}}$ = [Ba$^{2+}$]

5.1 × 10$^{-5}$ = [Ba$^{2+}$]

7. Calculate the wavelength (in nanometer) associated with a proton moving at 1.0 × 10$^3$ ms$^{-1}$

(Mass of proton = 1.67 × 10$^{-27}$ kg and $h = 6.63 \times 10^{-34}$ Js)

(1) 0.40 nm
(2) 2.5 nm
(3) 14.0 nm
(4) 0.032 nm

Sol: Ans [1]

$\lambda = \frac{h}{mv}$

$\lambda = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 10^3}$

$\lambda = 4 \times 10^{-10}$ meter = 0.4 nm

8. In context with the transition elements, which of the following statements is incorrect?

(1) In the highest oxidation states, the transition metal show basic character and form cationic complexes
(2) In the highest oxidation states of the first five transition elements (Sc to Mn), all the 4s and 3d electrons are used for bonding.
(3) Once the $d^5$ configuration is exceeded, the tendency to involve all the 3d electrons in bonding decreases
(4) In addition to the normal oxidation states, the zero oxidation state is also shown by these elements in complexes.

Sol: Ans [1]

In the highest oxidation state, the transition metal show acidic character
9. In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainty with which the position of the electron can be located is \( h = 6.6 \times 10^{-34} \text{ kg m}^2\text{s}^{-1} \), mass of electron, \( m = 9.1 \times 10^{-31} \text{ kg} \):

\[
\begin{align*}
\Delta v &= \frac{600 \times 0.005}{100} = 0.03 \text{ m/sec}^{-1} \\
\Delta x &= \frac{6.6 \times 10^{-34}}{0.03 \times 4 \times 3.14 \times 9.1 \times 10^{-31}} \\
\Delta x \cdot m \Delta x &= \frac{h}{4\pi} \\
\Delta v &= 1.92 \times 10^{-3} \text{ m}
\end{align*}
\]

Sol: Ans [2]

10. Which of the following pairs represents linkage isomers?

(1) \([\text{Pd}(\text{PPh}_3)_2\text{(NCS)}_2]\) and \([\text{Pd}(\text{PPh}_3)_2\text{(SCN)}_2]\)

(2) \([\text{CO(NH}_3)_5\text{NO}_3]\text{SO}_4\) and \([\text{CO(NH}_3)_5\text{SO}_4]\text{NO}_3\)

(3) \([\text{PtCl}_4(\text{NH}_3)_4]\text{Br}_2\) and \([\text{PtBr}_2(\text{NH}_3)_4]\text{Cl}_2\)

(4) \([\text{Cu(NH}_3)_4]\text{[PtCl}_4\) and \([\text{Pt(NH}_3)_4]\text{[CuCl}_4\]

Sol: Ans [1]

NCS is Ambident ligand

11. The bond dissociation energy of B – F in BF\(_3\) is 646 kJ mol\(^{-1}\) whereas that of C – F in CF\(_4\) is 515 kJ mol\(^{-1}\). The correct reason for higher B – F bond dissociation energy as compared to that of C – F is:

(1) stronger \(\sigma\) bond between B and F in BF\(_3\) as compared to that between C and F in CF\(_4\)

(2) significant \(\pi\) – \(\pi\) interaction between B and F in BF\(_3\) whereas there is no possibility of such interaction between C and F in CF\(_4\)

(3) lower degree of \(\pi\) – \(\pi\) interaction between B and F in BF\(_3\) than that between C and F in CF\(_4\)

(4) smaller size of B-atom as compared to that of C-atom.

Sol: Ans [2]

12. Using MO theory predict which of the following species has the shortest bond length?

(1) \(\text{O}_2^+\)

(2) \(\text{O}_2^-\)

(3) \(\text{O}_2^{2-}\)

(4) \(\text{O}_2^{3+}\)

Sol: Ans [4]

Bond order = 3, i.e. shortest bond length

13. A liquid was mixed with ethanol and a drop of concentrated H\(_2\)SO\(_4\) was added. A compound with a fruity smell was formed. The liquid was:

(1) HCHO

(2) \(\text{CH}_3\text{COCH}_3\)

(3) \(\text{CH}_3\text{COOH}\)

(4) \(\text{CH}_3\text{OH}\)

Sol: Ans [3]

\(\text{CH}_3\text{COOH} + \text{C}_2\text{H}_5\text{OH} \xrightarrow{\text{H}_2\text{SO}_4}\) \(\text{CH}_3\text{COOC}_2\text{H}_5 + \text{H}_2\text{O}\)

14. Which of the following on heating with aqueous KOH, produces acetaldehyde?

(1) \(\text{CH}_3\text{CH}_2\text{Cl}\)

(2) \(\text{CH}_2\text{CICH}_2\text{Cl}\)

(3) \(\text{CH}_3\text{CHCl}_2\)

(4) \(\text{CH}_3\text{COCl}\)
Sol: Ans [3]

\[ 
\text{CH}_3\text{CHCl}_2 + \text{aq. KOH} \underset{\text{unstable}}{\rightarrow} \text{CH}_3\text{CH(OH)}_2 \rightarrow \text{CH}_3\text{CHO} 
\]

15. Buna-N synthetic rubber is a copolymer of:

1. \( \text{H}_2\text{C} = \text{CH} = \text{CH}_2 \) and \( \text{H}_2\text{C} = \text{CH} = \text{CH}_2 \)
2. \( \text{H}_2\text{C} = \text{CH} - \text{CN} \) and \( \text{H}_2\text{C} = \text{CH} = \text{CH}_2 \)
3. \( \text{H}_2\text{C} = \text{CH} \) and \( \text{H}_2\text{C} = \text{CH} \)
4. \( \text{H}_2\text{C} = \text{CH} \) and \( \text{CH}_3 \)

Sol: Ans [2]

16. The two functional groups present in a typical carbohydrate are:

1. –CHO and –COOH
2. >\text{C} = \text{O} and –OH
3. –OH and –CHO
4. –OH and –COOH

Sol: Ans [2]

Typical carbohydrates are polyhydroxy carbonyl compounds.

17. In which of the following arrangements, the sequence is not strictly according to the property written against it?

1. \( \text{HF} < \text{HCl} < \text{HBC} < \text{HI} \): increasing acid strength
2. \( \text{NH}_3 < \text{PH}_3 < \text{AsH}_3 < \text{SbH}_3 \): increasing basic strength
3. \( \text{B} < \text{C} < \text{O} < \text{N} \): increasing first ionisation enthalpy
4. \( \text{CO}_2 < \text{SiO}_2 < \text{SnO}_2 < \text{PbO}_2 \): increasing oxidising power

Sol: Ans [2]

Among the hydrides of 15th group, basic strength decreases down the group.

18. A binary liquid solution is prepared by mixing n-heptane and ethanol. Which one of the following statements is correct regarding the behaviour of the solution?

1. The solution is non-ideal, showing +ve deviation from Raoult’s Law.
2. The solution is non-ideal, showing –ve deviation from Raoult’s law
3. n-heptane shows +ve deviation while ethanol shows –ve deviation from Raoult’s law
4. The solution formed is an ideal solution.

Sol: Ans [1]

19. The set representing the correct order of ionic radius is:

1. \( \text{Na}^+ > \text{Li}^+ > \text{Mg}^{2+} > \text{Be}^{2+} \)
2. \( \text{Li}^+ > \text{Na}^+ > \text{Mg}^{2+} > \text{Be}^{2+} \)
3. \( \text{Mg}^2 > \text{Be}^{2+} > \text{Li}^+ > \text{Na}^+ \)
4. \( \text{Li}^+ > \text{Be}^{2+} > \text{Na}^+ > \text{Mg}^{2+} \)
20. Arrange the carbanions, \((\text{CH}_3)_2\text{C}^-, \text{CCl}_3, (\text{CH}_3)_2\text{CH}, \text{C}_6\text{H}_5\text{CH}_2^-\), in order of their decreasing stability

\[
\begin{align*}
(1) & \quad (\text{CH}_3)_2\text{C} > \text{CCl}_3 > \text{C}_6\text{H}_5\text{CH}_2^- > (\text{CH}_3)_2\text{C} \\
(2) & \quad \text{CCl}_3 > \text{C}_6\text{H}_5\text{CH}_2^- > (\text{CH}_3)_2\text{C} > (\text{CH}_3)_2\text{C} \\
(3) & \quad (\text{CH}_3)_2\text{C} > (\text{CH}_3)_2\text{C} > \text{C}_6\text{H}_5\text{CH}_2^- > \text{CCl}_3 \\
(4) & \quad \text{C}_6\text{H}_5\text{CH}_2^- > (\text{CH}_3)_2\text{C} > (\text{CH}_3)_2\text{C} > \text{CCl}_3
\end{align*}
\]

21. Knowing that the Chemistry lanthanoids (Ln) is dominated by its +3 oxidation state, which of the following statements is incorrect?

\[
\begin{align*}
(1) & \quad \text{The ionic sizes of Ln (III) decrease in general with increasing atomic number.} \\
(2) & \quad \text{Ln (III) compounds are generally colourless.} \\
(3) & \quad \text{Ln (III) hydroxides are mainly basic in character} \\
(4) & \quad \text{Because of the large size of the Ln (III) ions the bonding in its compounds is predominantly ionic in character.}
\end{align*}
\]

22. The alkene that exhibits geometrical isomerism is:

\[
\begin{align*}
(1) & \quad \text{2-methyl propene} \\
(2) & \quad \text{2-butene} \\
(3) & \quad \text{2-methyl-2-butene} \\
(4) & \quad \text{propene}
\end{align*}
\]

23. The number of stereoisomers possible for a compound of the molecular formula \(\text{CH}_3 - \text{CH} = \text{CH} - \text{CH(OH)} - \text{Me}\) is:

\[
\begin{align*}
(1) & \quad 2 \\
(2) & \quad 4 \\
(3) & \quad 6 \\
(4) & \quad 3
\end{align*}
\]

24. In Cannizzaro reaction given below

\[
2\text{PhCHO} \xrightarrow{\text{OH}} \text{PhCH}_2\text{OH} + \text{PHCO}_2^-
\]

the slowest step is:

\[
\begin{align*}
(1) & \quad \text{the transfer of hydride to the carbonyl group} \\
(2) & \quad \text{the abstraction of proton from the carboxylic group} \\
(3) & \quad \text{the deprotonation of \text{PhCH}_2\text{OH}} \\
(4) & \quad \text{the attack of } \text{OH} \text{ at the carboxylic group}
\end{align*}
\]
25. On the basis of the following thermochemical data: \( \Delta G^0_{\text{H}_2\text{O}^0} = 0 \)

\[
\begin{align*}
\text{H}_2\text{O}(l) & \rightarrow \text{H}^+(aq) + \text{OH}^-(aq); \quad \Delta H = 57.32 \text{kJ} \\
\text{H}_2(g) + \frac{1}{2} \text{O}_2(g) & \rightarrow \text{H}_2\text{O}(l); \quad \Delta H = -286.20 \text{kJ}
\end{align*}
\]

The value of enthalpy of formation of OH\(^-\) ion at 25°C is:

(1) \(-228.88 \text{ kJ}\)  
(2) \(+228.88 \text{ kJ}\)  
(3) \(-343.52 \text{ kJ}\)  
(4) \(-22.88 \text{ kJ}\)

Sol: Ans [1]

26. Copper crystallises in fcc with a unit cell length of 361 pm. What is the radius of copper atom?

(1) 127 pm  
(2) 157 pm  
(3) 181 pm  
(4) 108 pm

Sol: Ans [1]

\[4r = \sqrt{2}a \quad r = \frac{361 \times \sqrt{2}}{4} = 127 \text{ pm}\]

27. In a fuel cell methanol is used as fuel and oxygen gas is used as an oxidizer. The reaction is

\[
\text{CH}_3\text{OH}(l) + \frac{3}{2} \text{O}_2(g) \rightarrow \text{CO}_2(g) + 2\text{H}_2\text{O}(l)
\]

At 298 K standard Gibb’s energies of formation for \(\text{CH}_3\text{OH}(l)\), \(\text{H}_2\text{O}(l)\) and \(\text{CO}_2(g)\) are \(-166.2\), \(-237.2\) and \(-394.4 \text{ kJ mol}^{-1}\) respectively. If standard enthalpy of combustion of methanol is 726 kJ mol\(^{-1}\), efficiency of the fuel cell will be:

(1) 87%  
(2) 90%  
(3) 97%  
(4) 80%

Sol: Ans [3]

\[
\begin{align*}
\Delta^0_g & = \sum \Delta^0_{g_{\text{product}}} - \sum \Delta^0_{g_{\text{reactant}}} \\
\Delta^0_g & = -702.2 \text{kJmol}^{-1} \quad \Delta^0_H = -726 \text{ kJ mole}^{-1} \\
\% \text{ efficiency} & = \frac{\Delta^0_g}{\Delta^0_H} \times 100 = 97%
\end{align*}
\]

28. Two liquids X and Y form an ideal solution. At 300 K, vapour pressure of the solution containing 1 mol of X and 3 mol of Y is 550 \(\text{mm Hg}\). At the same temperature, if 1 mol of Y is further added to this solution, vapour pressure of the solution increases by 10 \(\text{mm Hg}\). Vapour pressure (in \(\text{mm Hg}\)) of X and Y in their pure states will be, respectively:

(1) 300 and 400  
(2) 400 and 600  
(3) 500 and 600  
(4) 200 and 300

Sol: Ans [2]
\[ P_t = p_x^0 x + p_y^0 y \]

\[ 550 = p_x^0 \cdot \frac{1}{4} + p_y^0 \cdot \frac{3}{4} \] ... (i)

\[ 560 = p_x^0 \cdot \frac{1}{5} + p_y^0 \cdot \frac{4}{5} \] ... (ii)

By solving equations (i) and (ii)

\[ p_x^0 = 400 \text{ mm Hg} \]
\[ p_y^0 = 600 \text{ mm Hg} \]

29. Given

\[ E_{Fe^{3+}/Fe}^o = -0.036 \text{ V}, \quad E_{Fe^{2+}/Fe}^o = -0.439 \text{ V} \]

The value of standard electrode potential for the change \( Fe^{3+}_{(aq)} + e^- \rightarrow Fe^{2+}_{(aq)} \) will be

(1) 0.385V  \quad (2) 0.770V  \quad (3) 0.270V  \quad (4) -0.072V

Sol: Ans [2]

\[ Fe^{3+} + 3e^- \rightarrow Fe; \quad \Delta G_1^o = -3 \times F \times -0.036 \] ... (i)

\[ Fe^{2+} + 2e^- \rightarrow Fe; \quad \Delta G_2^o = -2 \times F \times -0.439 \] ... (ii)

By subtracting equation (ii) from equation (i)

We get \( E_{Fe^{3+}/Fe}^o + 0.77 \text{ V} \)

30. The half life period of a first order chemical reaction is 6.93 minutes. The time required for the completion of 99% of the chemical reaction will be \( (\log 2 = 0.301) \):

(1) 23.03 minutes  \quad (2) 46.06 minutes  \quad (3) 460.6 minutes  \quad (4) 230.3 minutes

Sol: Ans [2]

\[ \lambda = \frac{0.693}{t_{1/2}} \]

\[ \lambda = \frac{0.693}{6.93} = 10^{-1} \]

\[ t = \frac{2.303}{\lambda} \log \frac{a}{a-x} \]

\[ t = \frac{2.303}{10^{-1}} \log 100 \]

\[ t = 46.06 \text{ minutes} \]
PART-B: MATHEMATICS

Directions: Questions number 31 to 35 are Assertion - Reason type questions. Each of these questions contains two statements:

Statement - 1 (Assertion) and Statement - 2 (Reason)

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

31. Statement - 1: \( \sim(p \iff \sim q) \) is equivalent to \( p \iff q \).

Statement - 2: \( \sim(p \iff \sim q) \) is tautology.

1. Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
2. Statement - 1 is true, Statement - 2 is false.
3. Statement - 1 is false, Statement - 2 is true.
4. Statement - 2 is a correct explanation for Statement - 1.

Sol: Ans [2]

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Clearly, \( \sim(p \iff \sim q) \) is not a tautology because it does not contain \( T \) in the column of its truth table. Also, \( \sim(p \iff \sim q) \) and \( p \iff \sim q \) have the same truth value.

32. Let \( A \) be a \( 2 \times 2 \) matrix.

Statement - 1: \( \text{adj} (\text{adj} A) = A \)

Statement - 2: \( |\text{adj} A| = |A| \)

1. Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
2. Statement - 1 is true, Statement - 2 is false.
3. Statement - 1 is false, Statement - 2 is true.
4. Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.

Sol: Ans [1]

\[
\text{adj} (\text{adj} A) = |A|^{n-2} A
\]

Here \( n = 2 \)

\[
\Rightarrow \text{adj} (\text{adj} A) = |A|^0 A = A
\]

Also \( |\text{adj} A| = |A|^{n-1} \)

Here \( n = 2 \)

\[
\Rightarrow |\text{adj} A| = |A|^{2-1} = |A|.
\]
33. Let \( f(x) = (x + 1)^2 - 1, \ x \geq -1. \)

**Statement – 1 :**

The set \( \{ x : f(x) = f^{-1}(x) \} = \{0, -1\}. \)

**Statement – 2 :**

\( f \) is a bijection.

1. Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1.
2. Statement – 1 is true, Statement – 2 is false.
3. Statement – 1 is false, Statement – 2 is true.
4. Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1.

**Sol:** Ans [1]

\[ f(x) = x^3 + 2x \]
\[ f'(x) = 2x + 2 \geq 0 \quad (\text{since } x^3 - 1) \]

So, \( f(x) \) is increasing and continuous

So, \( f(x) \) is one-one

So, \( f \) is bijection

\[ f(x) = f^{-1}(x) \]
\[ \Rightarrow f(x) = x \]
\[ \Rightarrow x^2 + 2x = x \]
\[ \Rightarrow x(x + 1) = 0 \]
\[ \Rightarrow x = -1, 0 \]

34. **Statement – 1 :**

The variance of first \( n \) even natural numbers is \( \frac{n^2 - 1}{4} \).

**Statement – 2 :**

The sum of first \( n \) natural numbers \( \frac{n(n+1)}{2} \) and the sum of squares of first \( n \) natural numbers is \( \frac{n(n+1)(2n+1)}{6} \)

1. Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1.
2. Statement – 1 is true, Statement – 2 is false.
3. Statement – 1 is false, Statement – 2 is true.
4. Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1.

**Sol:** Ans [3]
Average of the numbers = \( \frac{2 + 4 + 6 + 8 + \ldots + 2n}{n} \)

\[ A = \frac{2}{n} \left( \frac{n(n+1)}{2} \right) = n + 1 \]

Variance = \( \frac{(A - 2)^2 + (A - 4)^2 + \ldots + (A - 2n)^2}{n} \)

\[ = \frac{A^2\Sigma 1 - 4A\Sigma n + 4\Sigma n^2}{n} = \frac{(n + 1)^2n - 2(n + 1)^2 + 4n(n + 1)(2n + 1)}{n} \]

\[ = (n + 1)^2 - 2(n + 1)^2 + \frac{2}{3}(n + 1)(2n + 1) \]

\[ = \frac{2}{3}(n + 1)(2n + 1) - (n + 1)^3 = (n + 1) \left[ \frac{4n + 2 - 3n - 3}{3} \right] = \frac{(n + 1)(n - 1)}{3} = \frac{n^2 - 1}{3} \]

35. Let \( f(x) = x|x| \) and \( g(x) = \sin x \).

**Statement – 1**: \( gof \) is differentiable at \( x = 0 \) and its derivative is continuous at that point.

**Statement – 2**: \( gof \) is twice differentiable at \( x = 0 \).

(1) Statement – 1 is true, Statement – 2 is true; Statement – 2 is not a correct explanation for Statement – 1.

(2) Statement – 1 is true, Statement – 2 is false

(3) Statement – 1 is false, Statement – 2 is true.

(4) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1.

**Sol:** Ans [2]

\( (gof) - g(f(x)) = g(x|x|) = \sin(x|x|) \)

At \( x = 0 \)

\[ \text{RHD.} = \lim_{h \to 0^+} \frac{(gof)(h) - (gof)(0)}{h} \]

\[ = \lim_{h \to 0^+} \frac{\sin(h|h|) - 0}{h} = \lim_{h \to 0^+} \frac{\sin h^2 \times h}{h^2} = 0 \]

\[ \text{LHD.} = \lim_{h \to 0^-} \frac{(gof)(-h) - (gof)(0)}{-h} \]

\[ = \lim_{h \to 0^-} \frac{\sin(-h^2) - 0}{-h} = 0 \]

So \( gof \) is differentiable at \( x = 0 \) and \( (gof)'(0) = 0 \)

\( (gof) = \sin x^2 \quad x \geq 0 \)

\( -\sin x^2 \quad x < 0 \)
\[(gof)'(x) = 2x \cos x^2 \quad x \geq 0\]
\[-2x \cos x^2 \quad x < 0\]

\[\lim_{x \to 0} (gof)'(x) = \lim_{x \to 0} 2x \cos x^2 = 0\]
\[\lim_{x \to 0} (gof)'(x) = \lim_{x \to 0} (-2x \cos x^2) = 0\]

So derivative of \(gof\) is continuous at \(x = 0\).

So statement - 1 is correct.

\[(gof)'(x) = 2x \cos x^2 \quad x \geq 0\]
\[-2x \cos x^2 \quad x < 0\]

At \(x = 0\)

\[RHD = \lim_{h \to 0^+} \frac{2h \cos h^2 - 0}{h} = 2\]
\[LHD = \lim_{h \to 0^-} \frac{(gof)(-h) - (gof)(0)}{-h} = \lim_{h \to 0^-} \frac{2h \cos h^2 - 0}{-h} = -2\]

So \(gof\) is not twice differentiable at \(x = 0\).

So statement - 2 is wrong.

36. The area of the region bounded by the parabola \((y - 2)^2 = x - 1\), the tangent to the parabola at the point \((2, 3)\) and the \(x\)-axis is

\[
\begin{align*}
& (1) \quad 6 & (2) \quad 9 & (3) \quad 12 & (4) \quad 3 \\
\end{align*}
\]

Sol: Ans [2]

\((y - 2)^2 = x - 1\)

Equation of tangent at \((2, 3)\) is

\[(y - 2)(3 - 2) = \frac{1}{3}(x + 2) - 1 \quad (T = 0)\]

\[\Rightarrow \quad y - 2 = \frac{x}{2}\]

Area bounded

\[
\begin{align*}
& = \int_{0}^{3} (x_1 - x_2) dy \\
& = \int_{0}^{3} ((y - 2)^2 + 1 - 2(y - 2)) dy \\
& = \int_{0}^{3} (y^2 - 6y + 4) dy = 9. \\
\end{align*}
\]
37. Given \( P(x) = x^4 + ax^3 + bx^2 + cx + d \) such that \( x = 0 \) is the only real root of \( P'(x) = 0 \). If \( P(-1) < P(1) \), then in the interval \([-1, 1]\):

1. \( P(-1) \) is not minimum but \( P(1) \) is the maximum of \( P \)
2. \( P(-1) \) is the minimum but \( P(1) \) is not the maximum of \( P \)
3. neither \( P(-1) \) is the minimum nor \( P(1) \) is the maximum of \( P \)
4. \( P(-1) \) is the minimum and \( P(1) \) is the maximum of \( P \)

Sol: Ans [1]

\[
P'(x) = 4x^3 + 3ax^2 + 2bx + c
\]

Since \( x = 0 \) is root of \( P'(x) = 0 \) \( \Rightarrow \) \( c = 0 \)

So, \( P'(x) = x(4x^2 + 3ax + 2b) \)

Either the roots of \( 4x^2 + 3ax + 2b \) are complex or real.

If roots are real, they are both equal to 0 [since \( x = 0 \) is the only real root of \( P'(x) = 0 \)]

\( \Rightarrow a = b = 0 \)

So, \( P(x) = x^4 + d \)

\( \Rightarrow P(-1) = P(1) \) which is wrong. [Since \( P(-1) < P(1) \)]

So, roots of \( 4x^2 + 3ax + 2b \) are complex.

\( \Rightarrow 4x^2 + 3ax + 2b > 0 \quad \forall \ x \in \mathbb{R} \)

when \( x < 0 \), \( P'(x) < 0 \) and when \( x > 0 \), \( P'(x) > 0 \)

So, \( x = 0 \) is minima.

So graph of \( P(x) \) is as shown below.

From the graph, \( P(-1) \) is not minimum but \( P(1) \) is maximum of \( P \).

38. The shortest distance between the line \( y - x = 1 \) and the curve \( x = y^2 \) is

\[
\begin{align*}
(1) & \quad \frac{2\sqrt{3}}{8} \\
(2) & \quad \frac{3\sqrt{3}}{5} \\
(3) & \quad \frac{\sqrt{3}}{4} \\
(4) & \quad \frac{3\sqrt{5}}{8}
\end{align*}
\]

Sol: Ans [4]

Shortest distance always occurs along common normal.

The normal to \( y^2 = x \) at \( A\left(\frac{1}{4}, \frac{1}{2}\right) \) is \( y = -tx + \frac{t^2}{2} + \frac{t^3}{4} \)

Since this normal should be perpendicular to \( y = x + 1 \), its slope = -1

\( \Rightarrow -t = -1 \quad \Rightarrow \quad t = 1 \)

So, the point \( A \equiv \left(\frac{1}{4}, \frac{1}{2}\right) \)

Perpendicular distance of \( \left(\frac{1}{4}, \frac{1}{2}\right) \) from \( y = x + 1 \)

\[
\frac{1}{4}\frac{1}{2} - \frac{1}{2} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}
\]
39. Let the line \( \frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2} \) lie in the plane \( x + 3y - \alpha z + \beta = 0 \). Then \((\alpha, \beta)\) equals

(1) \((-6, 7)\)
(2) \((5, -15)\)
(3) \((-5, 5)\)
(4) \((6, -17)\)

**Sol:** Ans [1]

Since the line lies in the plane, the line is perpendicular to the normal of plane.

\[ 3 \times 1 - 5 \times 3 - 2 \alpha = 0 \]
\[ \alpha = -6 \]

Also the point \((2, 1, -2)\) lies in the plane.

\[ 2 + 3 \times 1 + 6 \times -2 + \beta = 0 \]
\[ \beta = 7 \]

40. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is

(1) at least 500 but less than 750
(2) at least 750 but less than 1000
(3) at least 1000
(4) less than 500

**Sol:** Ans [3]

4 novels can be selected in \( ^6C_4 \) ways and 1 dictionary can be selected in \(^3C_1\) ways.

The place of dictionary is fixed and 4 novels can be arranged in 4! ways.

Total number of arrangements = \(^6C_4 \times ^3C_1 \times 4!\)

\[ \quad = 1080. \]

41. In a binomial distribution \( B\left( n, \ p = \frac{1}{4} \right) \), if the probability of at least one success is greater than or equal to \( \frac{9}{10} \), then \( n \) is greater than:

(1) \( \frac{1}{\log_{10}^4 + \log_{10}^5} \)
(2) \( \frac{9}{\log_{10}^4 - \log_{10}^6} \)
(3) \( \frac{4}{\log_{10}^4 - \log_{10}^6} \)
(4) \( \frac{1}{\log_{10}^4 - \log_{10}^6} \)

**Sol:** Ans [4]

Probability of atleast one success

\[ = 1 - \text{Probability of no success} \]

\[ = 1 - \left( \frac{3}{4} \right)^n \geq \frac{9}{10} \]

\[ \Rightarrow \left( \frac{3}{4} \right)^n \leq \frac{1}{10} \]

\[ \Rightarrow \left( \frac{4}{3} \right)^n \geq 10 \]

Taking log, \( n \log \frac{4}{3} \geq \log 10 \)
42. The lines \( p(p^2 + 1)x - y + q = 0 \) and \( (p^2 + 1)x + (p^2 + 1)y + 2q = 0 \) are perpendicular to a common line for:

- (1) exactly one value of \( p \)
- (2) exactly two values of \( p \)
- (3) more than two values of \( p \)
- (4) no value of \( p \)

**Sol:** Ans [1]

Since the two lines are perpendicular to common line, the two lines must be parallel. So their slopes are equal.

\[
P(p^2 + 1) = \frac{-(p^2 + 1)^2}{p^2 + 1}
\]

\[
\Rightarrow (p^2 + 1)(p + 1) = 0
\]

\[
\Rightarrow p = -1.
\]

43. If \( A, B \) and \( C \) are three sets such that \( A \cap B = A \cap C \) and \( A \cup B = A \cup C \), then:

- (1) \( A = C \)
- (2) \( B = C \)
- (3) \( A \cap B = \emptyset \)
- (4) \( A = B \)

**Sol:** Ans [2]

\[
A \cup B = A \cup C
\]

\[
(A \cup B) \cap B = (A \cup C) \cap B
\]

\[
B = (A \cap B) \cup (C \cap B)
\]

\[
= (A \cap C) \cup (A \cap C)
\]

Again \( A \cup C = A \cup B \)

\[
(A \cup C) \cap C = (A \cup B) \cap C
\]

\[
C = (A \cap C) \cup (B \cap C)
\]

From (1) & (2)

\[
B = C.
\]

44. For real \( x \), let \( f(x) = x^3 + 5x + 1 \), then

- (1) \( f \) is onto \( \mathbb{R} \) but not one-one
- (2) \( f \) is one-one and onto \( \mathbb{R} \)
- (3) \( f \) is neither one-one nor onto \( \mathbb{R} \)
- (4) \( f \) is one-one but not onto \( \mathbb{R} \)

**Sol:** Ans [2]

Since \( f \) is odd-degree polynomial, \( f \) is onto.

\[
f'(x) = 3x^2 + 5 > 0 \quad \forall \quad x \in \mathbb{R}
\]

So, \( f(x) \) is continuous increasing function.

\[
\Rightarrow f \text{ is one-one.}
\]
45. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where $c_1$ and $c_2$ are arbitrary constants, is

1. $y'' = y'$
2. $yy' = y$
3. $yy'' = (y')^2$
4. $y' = y^2$

Sol: Ans [3]

\[ y = c_1 e^{c_2 x} \]
\[ y' = c_1 c_2 e^{c_2 x} \]
\[ y'' = c_1 c_2^2 e^{c_2 x} \]

\[ \Rightarrow \frac{y''}{y} = \frac{y'}{y} \Rightarrow yy'' = (y')^2 \]

46. Let $a, b, c$ be such that $b(a + c) \neq 0$. If

\[
\begin{vmatrix}
 a & a+1 & a-1 \\
 -b & b+1 & b-1 \\
 c & c-1 & c+1 \\
\end{vmatrix}
+ \begin{vmatrix}
 a+1 & b+1 & c-1 \\
 a-1 & b-1 & c+1 \\
 (-1)^{n+2} a & (-1)^{n+1} b & (-1)^n c \\
\end{vmatrix} = 0
\]

then the value of $n$ is

1. any even integer
2. any odd integer
3. any integer
4. zero

Sol: Ans [2]

\[
\begin{vmatrix}
 a & a+1 & a-1 \\
 -b & b+1 & b-1 \\
 c & c-1 & c+1 \\
\end{vmatrix} = \begin{vmatrix}
 a+1 & b+1 & c-1 \\
 a-1 & b-1 & c+1 \\
 (-1)^{n+2} a & (-1)^{n+1} b & (-1)^n c \\
\end{vmatrix}
\]

Adding 2nd determinant,

\[
\begin{vmatrix}
 a+1 & b+1 & c-1 \\
 a-1 & b-1 & c+1 \\
 a & -b & c \\
\end{vmatrix}
= \begin{vmatrix}
 a+1 & b+1 & c-1 \\
 a-1 & b-1 & c+1 \\
 a((-1)^n + 1) & -b(1+(-1)^n) & c((-1)^n + 1) \\
\end{vmatrix}
\]

\[
=((-1)^n + 1) \begin{vmatrix}
 a+1 & b+1 & c-1 \\
 a-1 & b-1 & c+1 \\
 a & -b & c \\
\end{vmatrix}
\]

\[ R_1 \rightarrow R_1 - R_2 \]
\[ \begin{vmatrix} 2 & 2 & -2 \\ a - 1 & b - 1 & c + 1 \\ a & -b & c \end{vmatrix} \]

\[ c_1 \rightarrow c_1 + c_3, \quad c_2 \rightarrow c_2 + c_3 \]

\[ \begin{vmatrix} 0 & 0 & -2 \\ a + c & b + c & c + 1 \\ a + c & c - b & c \end{vmatrix} \]

\[ = ((-1)^n + 1)(a + c)(-2)(c - b - b - c) \]

\[ = 4((-1)^n + 1)(a + c)b = 0 \]

\[ \Rightarrow (-1)^n + 1 = 0 \quad \text{(Since } b(a + c) \neq 0) \]

\[ \Rightarrow n \text{ is any odd integer.} \]

47. The remainder left out when \(8^{2n} - (62)^{2n+1}\) is divided by 9 is

(1) 2  (2) 7  (3) 8  (4) 0

Sol: Ans [1]

\[ 8^{2n} - (62)^{2n+1} = (9 - 1)^{2n} - (9 \times 7 - 1)^{2n+1} = 9K + (-1)^{2n} - (9l + (-1)^{2n+1}) \]

\[ = 9K + 1 - 9l - (-1) = 9t + 2 \]

So remainder is 2.

48. Let \(y\) be an implicit function of \(x\) defined by \(x^2 - 2x^3 \cot y - 1 = 0\). Then \(y'(1)\) equals

(1) 1  (2) \log 2  (3) -\log 2  (4) -1

Sol: Ans [4]

\[ x^2 - 2x^3 \cot y - 1 = 0 \quad \ldots (1) \]

Differentiating w.r.t. \(x\),

\[ x^{2}\left(2 + 2\log x\right) - 2x^{3} \cot y \left(1 + \log x - \frac{1}{\cos y \sin y} \frac{dy}{dx}\right) = 0 \]

(Take \(z = x^3 \cot y \Rightarrow \log z = x^3 \log x + \log (\cot y)\) and now differentiate both sides)

Putting \(x = 1,\)

\[ 2 - 2 \cot y \left(1 - \frac{1}{\cos y \sin y} \frac{dy}{dx}\right) = 0 \]

\[ \Rightarrow 2 - 2 \cot y + \frac{2}{\sin^2 y} \frac{dy}{dx} = 0 \quad \ldots (2) \]

Putting \(x = 1\) in (1),

\[ 1 - 2 \cot y - 1 = 0 \Rightarrow \cot y = 0 \Rightarrow \csc^2 y = 1 + \cot^2 y = 1 \]

So, (2) becomes

\[ 2 + \frac{2dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -1 \]
49. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of $x$, the expression $3b^2x^2 + 6bcx + 2c^2$ is

(1) less than $4ab$  
(2) greater than $-4ab$
(3) less than $-4ab$  
(4) greater than $4ab$

Sol: Ans [2]

Since roots of $bx^2 + cx + a = 0$ are imaginary, its discriminant $< 0$

$D = c^2 - 4ab < 0$  \[\Rightarrow c^2 < 4ab\]

The graph of $3b^2x^2 + 6bcx + 2c^2$ will be upward parabola. So its minimum value will be $\frac{-D}{4a}$

$$\frac{-D}{4a} = \frac{-9(36b^2c^2 - 24b^2c^2)}{12b^2} = -c^2$$

$c^2 < 4ab$  \[\Rightarrow -c^2 > -4ab\]

So $3b^2x^2 + 6bcx + 2c^2 \geq -c^2 > -4ab$

50. The sum to infinity of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} \ldots$ is

(1) 3  
(2) 4  
(3) 6  
(4) 2

Sol: Ans [1]

$$S_\infty = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} \ldots$$

$$\frac{1}{3}S_\infty = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} \ldots$$

$$\frac{2}{3}S_\infty = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} \ldots$$

$$\frac{2}{3}S_\infty = 1 + \frac{1}{3} + \frac{4}{9} \left(1 + \frac{1}{3} + \frac{1}{3^2} + \ldots \infty\right) = 1 + \frac{1}{3} + \left(\frac{4 \times \frac{1}{3}}{1 - \frac{1}{3}}\right) = 1 + \frac{4 \times \frac{3}{2}}{2} = 1 + \frac{2}{3} + \frac{2}{3} = 2$$

$$S_\infty = \frac{2 \times \frac{3}{2}}{2} = \frac{3}{2}$$

51. The projections of a vector on the three coordinate axis are 6, $-3$, 2 respectively. The direction cosines of the vector are

(1) $\frac{6}{5}, -\frac{3}{5}, \frac{2}{5}$  
(2) $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$  
(3) $-\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$  
(4) 6, $-3$, 2
Sol: Ans [2]

The vector is \( \vec{r} = 6\hat{i} - 3\hat{j} + 2\hat{k} \)

So, \( |\vec{r}| = \sqrt{6^2 + 3^2 + 2^2} = 7 \)

So, \( \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k} \)

So direction cosines are \( \frac{6}{7}, \frac{-3}{7}, \frac{2}{7} \)

52. Let A and B denote the statements

A : \( \cos \alpha + \cos \beta + \cos \gamma = 0 \)

B : \( \sin \alpha + \sin \beta + \sin \gamma = 0 \)

If \( \cos (\beta - \gamma) + \cos (\gamma - \alpha) + \cos (\alpha - \beta) = \frac{3}{2} \), then :

(1) A is false and B is true

(2) both A and B are true

(3) both A and B are false

(4) A is true and B is false

Sol: Ans [2]

Let \( \cos \alpha + \cos \beta + \cos \gamma = x \)

and \( \sin \alpha + \sin \beta + \sin \gamma = y \)

Then \( x^2 + y^2 = (\cos \alpha + \cos \beta + \cos \gamma)^2 + (\sin \alpha + \sin \beta + \sin \gamma)^2 \)

\( = 3 + 2 \Sigma (\cos \alpha \cos \beta + \sin \alpha \sin \beta) \)

\( = 3 + 2 \Sigma \cos (\alpha - \beta) \)

\( = 3 + 2 [\cos (\alpha - \beta) + \cos (\beta - \gamma) + \cos (\gamma - \alpha)] \)

\( = 3 + 2 \times \frac{-3}{2} = 0 \)

\( \Rightarrow x = y = 0 \)

53. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals :

(1) \( \frac{1}{7} \)

(2) \( \frac{5}{14} \)

(3) \( \frac{1}{50} \)

(4) \( \frac{1}{14} \)

Sol: Ans [4]

Since product of digits is zero, then one digit must be 0.

Let A be the event of getting a ticket whose one digit is 0.

A = \{00, 01, ..., 09, 10, 20, 30, 40\}

B be the event of getting product of digits is 8.

B = \{08, 18, 24, 42\} . \ A \cap B = \{08\}

Now, \( P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1}{50} \times \frac{1}{14} = \frac{1}{14} \)
54. Three distinct points $A$, $B$ and $C$ are given in the 2-dimensional coordinate plane such that the ratio of
the distance of any one of them from the point $(1, 0)$ to the distance from the point $(-1, 0)$ is equal to
\[ \frac{1}{3} \]
Then the circumcentre of the triangle $ABC$ is at the point:

(1) \[ \left( \frac{5}{4}, 0 \right) \]  
(2) \[ \left( \frac{5}{2}, 0 \right) \]  
(3) \[ \left( \frac{5}{3}, 0 \right) \]  
(4) \[ (0, 0) \]

Sol: **Ans [1]**

Let $D \equiv (1, 0)$ and $E \equiv (-1, 0)$

Let $P \equiv (x, y)$

\[ \frac{PD}{PE} = \frac{1}{3} \]

\[ \Rightarrow 3\sqrt{(x-1)^2 + y^2} = \sqrt{(x+1)^2 + y^2} \]

\[ \Rightarrow 9(x^2 + y^2 - 2x + 1) = x^2 + y^2 + 2x + 1 \]

\[ \Rightarrow 8(x^2 + y^2) - 20x + 8 = 0 \]

\[ \Rightarrow x^2 + y^2 = \frac{5}{2}x + 1 \]

So locus of point $P$ is this circle and 3 points $A, B, C$ lie on this circle.

So circumcentre of $\Delta ABC = \text{centre of above circle} = \left( \frac{5}{4}, 0 \right)$.

55. If the mean deviation of the numbers $1$, $1 + d$, $1 + 2d$, …., $1 + 100d$ from their mean is 255, then the $d$
is equal to:

(1) 20.0  
(2) 10.1  
(3) 20.2  
(4) 10.2

Sol: **Ans [2]**

\[ \bar{x} = \frac{1 + (1 + d) + (1 + 2d) + ... + (1 + 100d)}{101} \]

\[ \bar{x} = \frac{01 + d(1 + 2 + ... + 100)}{101} \]

\[ = \frac{101 + 50d \times 101}{101} \]

\[ = 1 + 50d \]

\[ M.D.(\bar{x}) = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + ... + |x_n - \bar{x}|}{n} \]

or \[ 255 = \frac{|1-1-50d| + |1+d-1-50d| + ... + |1+100d-1-50d|}{101} \]

or \[ 255 = \frac{50d + 49d + ... + d + 0 + d + ... + 50d}{101} = \frac{2d(1 + 2 + 3 + ... + 50)}{101} \]
56. The ellipse \( x^2 + 4y^2 = 4 \) is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point \((4, 0)\). Then the equation of the ellipse is:

- (1) \( x^2 + 12y^2 = 16 \)
- (2) \( 4x^2 + 48y^2 = 48 \)
- (3) \( 4x^2 + 64y^2 = 48 \)
- (4) \( x^2 + 16y^2 = 16 \)

**Sol:** Ans [1]

The ellipse will pass through \((4, 0)\) and \((2, 1)\) hence \( x^2 + 12y^2 = 16 \).

57. If \( \frac{4}{z} = 2 \), then the maximum value of \( |z| \) is equal to

- (1) \( \sqrt{5} + 1 \)
- (2) \( 2 \)
- (3) \( 2 + \sqrt{2} \)
- (4) \( \sqrt{3} + 1 \)

**Sol:** Ans [1]

\[
\begin{align*}
|z_1 - z_2| & \geq ||z_1| - |z_2|| \\
|z - \frac{4}{z}| & \geq |z| - \left| \frac{4}{z} \right| \\
\Rightarrow \quad -2 & \leq |z| - \left| \frac{4}{z} \right| \leq 2 \\
|z| & \in [\sqrt{5} - 1, \sqrt{5} + 1]
\end{align*}
\]

58. If \( P \) and \( Q \) are the points of intersection of the circles \( x^2 + y^2 + 3x + 7y + 2p - 5 = 0 \) and \( x^2 + y^2 + 2x + 2y - p^2 = 0 \), then there is a circle passing through \( P, Q \) and \((1, 1)\) for

- (1) all except one value of \( p \)
- (2) all except two values of \( p \)
- (3) exactly one value of \( p \)
- (4) all values of \( p \)

**Sol:** Ans [1]

The radical axis of two circles is \( S_1 - S_2 = 0 \).

\[
\Rightarrow \quad x + 5y + 2p - 5 + p^2 = 0
\]

If radical axis passes through \((1, 1)\),

\[
\Rightarrow \quad 1 + 5 + 2p - 5 + p^2 = 0
\]

\[
\Rightarrow \quad p^2 + 2p + 1 = 0
\]

\[
\Rightarrow \quad p = -1
\]

So for \( p = -1 \), radical axis will pass through \((1, 1)\).

\[
\Rightarrow \quad P, Q \text{ and } (1, 1) \text{ will become collinear.}
\]

So, no circle will pass through \( P, Q \) and \((1, 1)\).

For all other values of \( p \), points \( P, Q \) and \((1, 1)\) will be non-collinear and thereby lie on a circle.

So points \( P, Q \) and \((1, 1)\) will lie on a circle for all values of \( p \) except \( p = -1 \).
59. If \( \vec{u}, \vec{v}, \vec{w} \) are non-coplanar vectors and \( p, q \) are real numbers, then the equality
\[
[3\vec{u} \quad p\vec{v} \quad p\vec{w}] - [p\vec{v} \quad \vec{w} \quad q\vec{u}] - [2\vec{w} \quad q\vec{v} \quad q\vec{u}] = 0
\]
holds for:
- (1) exactly two values of \( (p, q) \)
- (2) more than two but not all values of \( (p, q) \)
- (3) all values of \( (p, q) \)
- (4) exactly one value of \( (p, q) \)

**Sol:** Ans [4]

\[
[3\vec{u} \quad p\vec{v} \quad p\vec{w}] - [p\vec{v} \quad \vec{w} \quad q\vec{u}] - [2\vec{w} \quad q\vec{v} \quad q\vec{u}] = 3p^2[\vec{u} \quad \vec{v} \quad \vec{w}] - pq[\vec{v} \quad \vec{w} \quad \vec{u}] - 2q^2[\vec{w} \quad \vec{v} \quad \vec{u}] \\
= 3p^2[\vec{u} \quad \vec{v} \quad \vec{w}] - pq[\vec{u} \quad \vec{v} \quad \vec{w}] + 2q^2[\vec{u} \quad \vec{v} \quad \vec{w}] \\
= (3p^2 - pq + 2q^2)[\vec{u} \quad \vec{v} \quad \vec{w}] = 0 \\
\Rightarrow 3p^2 - pq + 2q^2 = 0 \quad (\text{since } [\vec{u} \quad \vec{v} \quad \vec{w}] \neq 0) \\
\text{Its discriminant } = q^2 - 24q^2 = -23q^2 \leq 0 \\
\text{For real roots, } D = 0 \Rightarrow q = 0 \Rightarrow p = 0 \\
\text{So, equation holds for exactly one value of } (p, q) \text{ i.e. } (0, 0)
\]

60. \( \int_0^\pi \cot x \, dx \), where \([.]\) denotes the greatest integer function, is equal to:

- (1) 1
- (2) \(-1\)
- (3) \(-\frac{\pi}{2}\)
- (4) \(\frac{\pi}{2}\)

**Sol:** Ans [3]

\[
\int_0^\pi \cot x \, dx = \int_0^{\pi/2} \cot x \, dx + \int_{\pi/2}^\pi \cot x \, dx \\
\text{In 2}^{\text{nd}} \text{ integral, put } x = \pi - y \\
\int_{\pi/2}^\pi \cot x \, dx = - \int_0^{\pi/2} [-\cot y] \, dy = \int_0^{\pi/2} [-\cot x] \, dx \\
\text{So, } \int_0^\pi \cot x \, dx = \int_0^{\pi/2} \cot x \, dx + \int_0^{\pi/2} [-\cot x] \, dx = \int_0^{\pi/2} ([\cot x] + [-\cot x]) \, dx \\
\text{wherever } \cot x \in I, [\cot x] + [-\cot x] = 0 \text{ but such points will be solitary and thereby will not affect the area under } [\cot x] + [-\cot x]. \\
\text{At all other points, } [\cot x] + [-\cot x] = -1 \\
\text{So, } \int_0^{\pi/2} ([\cot x] + [-\cot x]) \, dx = \int_0^{\pi/2} (-1) \, dx = -\frac{\pi}{2}.
\]
61. Consider a rubber ball freely falling from a height \( h = 4.9\) m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be

\[
\begin{align*}
(1) & \quad v = v_1 - \frac{v_1}{2} - \frac{v_1}{4} - \cdots \\
(2) & \quad v = v_1 - \frac{v_1}{2} - \frac{v_1}{4} - \cdots \\
(3) & \quad v = v_1 - \frac{v_1}{2} - \frac{v_1}{4} - \cdots \\
(4) & \quad v = v_1 - \frac{v_1}{2} - \frac{v_1}{4} - \cdots 
\end{align*}
\]

Sol: Ans [2]

\[
v = 0 - gt
\]

since initial velocity is zero.

\[
\therefore \quad v = -gt
\]

after collision its direction become positive

62. The height at which the acceleration due to gravity becomes \( g/9 \) (where \( g \) = the acceleration due to gravity on the surface of the earth) in terms of \( R \), the radius of the earth is

\[
\begin{align*}
(1) & \quad \frac{R}{\sqrt{2}} \\
(2) & \quad \frac{R}{2} \\
(3) & \quad \sqrt{2R} \\
(4) & \quad 2R
\end{align*}
\]

Sol: Ans [4]

\[
g_h = g \left( \frac{R}{R + h} \right)^2
\]

\[
\frac{g}{9} = g \left( \frac{R}{R + h} \right)^2
\]

\[
\Rightarrow \quad \frac{R + h}{R} = 3 \Rightarrow h = 2R
\]
63. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature \( \theta \) along the length \( x \) of the bar from its hot end is best described by which of the following figures?

![Figure 1](image1.png)

![Figure 2](image2.png)

![Figure 3](image3.png)

![Figure 4](image4.png)

**Sol:** Ans [1]

In steady state, \( \frac{d\theta}{dx} = \text{constant} \) (–ve)

64. Two points P and Q are maintained at the potentials of 10V and –4V, respectively. The work done in moving 100 electrons from P to Q is

(1) \( 9.60 \times 10^{-17} \) J  
(2) \( -2.24 \times 10^{-16} \) J  
(3) \( 2.24 \times 10^{-16} \) J  
(4) \( -9.60 \times 10^{-17} \) J

**Sol:** Ans [3]

\[
W = q \Delta V \\
= (-1.6 \times 10^{-19} \times 100) (-4 - 10) \\
= 2.24 \times 10^{-16} \text{ J}
\]

**Directions:** Question number 65 and 66 are based on the following paragraph:

A current loop ABCD is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = \( b \)) and DA (radius = \( a \)) of the loop are joined by two straight wires AB and CD. A steady current \( I \) is flowing in the loop. Angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current \( I_1 \) following out of the plane of the paper is kept at the origin.

65. The magnitude of the magnetic field (B) due to the loop ABCD at the origin (O) is:

(1) \( \frac{\mu_0 I(b-a)}{24ab} \)  
(2) \( \frac{\mu_0 I}{4\pi} \left[ b-a \right] \)  
(3) \( \frac{\mu_0 I}{4\pi} \left[ 2(b-a) + \pi/3(a+b) \right] \)  
(4) zero

**Sol:** Ans [1]

\[
B = \frac{\mu_0 I}{4\pi} \times 0 \left[ \frac{1}{a} - \frac{1}{b} \right] \\
= \frac{\mu_0 I}{6} \left( \frac{1}{a} - \frac{1}{b} \right) \\
= \frac{\mu_0 I}{24} \left[ b-a \right] \]
66. Due to the presence of the current $I_1$ at the origin:

1. The forces on AD and BC are zero
2. The magnitude of the net force on the loop is given by $\frac{I_1 I}{4\pi} \mu_0 [2(b - a) + \pi/3(a + b)]$
3. The magnitude of the net force on the loop is given by $\frac{\mu_0 I_1}{24ab} (b - a)$
4. The force on AB and DC are zero

Sol: Ans [1]

Field due to I is parallel to DA and Antiparallel to BC hence force is zero

Directions: Question number 67 and 68 and 69 are based on the following paragraph:

Two moles of helium gas are taken over the cycle ABCDA, as shown in the P – T diagram.

67. Assuming the gas to be ideal the work done on the gas in taking it from A to B is

1. $300 \, R$
2. $400 \, R$
3. $500 \, R$
4. $200 \, R$

Sol: Ans [2]

A $\rightarrow$ B: process is isobaric

$W = P (V_B - V_A)$

$= nR (T_B - T_A)$

$= 2 \times R (500 - 300)$

$= 400 \, R$

68. The work done on the gas in taking it from D to A is

1. $+ 414 \, R$
2. $- 690 \, R$
3. $+ 690 \, R$
4. $- 414 \, R$

Sol: Ans [4]

D $\rightarrow$ A: isothermal process

$W = nRT \ln \left( \frac{P_1}{P_2} \right)$

$= 2R \times 300 \ln \left( \frac{1}{2} \right)$

$= - 600 \, R \ln 2$

$= - 414 \, R$
69. The net work done on the gas in the cycle ABCDA is

(1) 276 R  (2) 1076 R  (3) 1904 R  (4) zero

Sol: Ans [1]

\[ W_{AB} = 400 \, \text{R} \]
\[ W_{CD} = P (V_D - V_C) \]
\[ = nR (T_D - T_C) \]
\[ = 2 \times R (300 - 500) \]
\[ = -400 \, \text{R} \]
\[ W_{DA} = -600 \, \text{R ln2} \]
\[ W_{BC} = nR(500) \ln \left( \frac{2}{1} \right) \]
\[ = 1000 \, \text{R ln2} \]
\[ \therefore \text{net work} = R \ln 2 (1000 - 600) \]
\[ = 400 \, \text{R ln2} \]
\[ = 276 \, \text{R} \]

70. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half -a-degree (= 0.5°), then the least count of the instrument is

(1) half minute  (2) one degree  (3) half degree  (4) one minute

Sol: Ans [4]

Least count = \[ \frac{0.5}{30} \times 60 \, \text{minute} \]
\[ = 1 \, \text{minute} \]

71. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the two corners. If the net electrical force on Q is zero, then Q/q equals.

(1) -1  (2) 1  (3) \[ \frac{-1}{\sqrt{2}} \]  (4) \[ -2\sqrt{2} \]

Sol: Ans [4]

Force on Q is zero
\[ \Rightarrow \frac{kQ^2}{2a^2} + \frac{kQq}{a^2} \sqrt{2} = 0 \]
\[ \Rightarrow \frac{Q}{q} = -2\sqrt{2} \]
72. One kg of a diatomic gas is at a pressure of $8 \times 10^4$ N/m$^2$. The density of the gas is 4 kg/m$^3$. What is the energy of the gas due to its thermal motion?

(1) $5 \times 10^4$ J  
(2) $6 \times 10^4$ J  
(3) $7 \times 10^4$ J  
(4) $3 \times 10^4$ J  

**Sol:** Ans [1]  

$$E = \frac{1}{2} m v_{rms}^2 = \frac{5}{2} p V = \frac{5}{2} p \frac{m}{\rho} = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 \text{ J}$$

73. An inductor of inductance $L = 400 \text{ mH}$ and resisters of resistances $R_1 = 2 \Omega$ and $R_2 = 2 \Omega$ are connected to a battery of emf $12 \text{ V}$ as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is

(1) $\frac{12}{t} e^{-3t} \text{ V}$  
(2) $6(1 - e^{-t/0.2}) \text{ V}$  
(3) $12 e^{-5t} \text{ V}$  
(4) $6 e^{-5t} \text{ V}$  

**Sol:** Ans [3]  

$$i = \frac{E}{R_2}(1 - e^{-t/\tau})$$  

$$\therefore \quad V_L = -L \frac{di}{dt} = -6e^{-t/\tau}$$  

$$|V_L| = 12e^{-t/\tau}$$  

$$\tau = \frac{L}{R_2}$$

This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements:

74. **Statement-1:** The temperature dependence of resistance is usually given as $R = R_0 (1 + \alpha \Delta T)$. The resistance of a wire change from $100 \Omega$ to $150 \Omega$ when its temperature is increased from $27^\circ \text{C}$ to $227^\circ \text{C}$. This implies that $\alpha = 2.5 \times 10^{-3}/^\circ \text{C}$.

**Statement-2:** $R = R_0 (1 + \alpha \Delta T)$ is valid only when the change in the temperature $\Delta T$ is small and $\Delta R = (R - R_0) << R_0$.

(1) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1
(2) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true
(4) Statement-1 is true, Statement-2 is false

Sol: Ans [3]

The formula \( R = R_0 (1 + \alpha \Delta t) \) is not for large charge in temperature.

75. The transition from the state \( n = 4 \) to \( n = 3 \) in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from:

(1) \( 3 \rightarrow 2 \)  
(2) \( 4 \rightarrow 2 \)  
(3) \( 5 \rightarrow 4 \)  
(4) \( 2 \rightarrow 1 \)

Sol: Ans [3]

In higher energy orbits, energy gap between two consecutive level decreases. Therefore wavelength of emitted photon increases.

76. A mixture of light consisting of wavelength 590 nm and an unknown wavelength, illuminates Young’s double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is:

(1) 885.0 nm  
(2) 442.5 nm  
(3) 776.8 nm  
(4) 393.4 nm

Sol: Ans [2]

\[
\frac{3 \times 590 \times D}{d} = \frac{4 \times \lambda \times D}{d}
\]

\[
\Rightarrow \lambda = 442.5 \text{ nm}
\]

77. A particle has an initial velocity of \( 3\hat{i} + 4\hat{j} \) and an acceleration of \( 0.4\hat{i} + 0.3\hat{j} \). Its speed after 10s is:

(1) 7\sqrt{2} units  
(2) 7 units  
(3) 8.5 units  
(4) 10 units

Sol: Ans [1]

\[
v = (3\hat{i} + 4\hat{j}) + (0.4 \hat{i} + 0.3 \hat{j}) \times 10
\]

\[
= 7\hat{i} + 7\hat{j}
\]

\[
\Rightarrow \text{Speed} = 7\sqrt{2} \text{ m/sec}
\]

78. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is \( (hc = 1240 \text{ eV.nm}) \)

(1) 1.41 eV  
(2) 1.51 eV  
(3) 1.68 eV  
(4) 3.09 eV

Sol: Ans [1]

Energy of photon = \( \frac{1240}{400} = 3.1 \text{ eV} \)

Using Einstein’s equation

\[
\phi = 3.1 - 1.68 \approx 1.41 \text{ eV}
\]
79. Three sound waves of equal amplitudes have frequencies \((v-1), v, (v+1)\). They superpose to give beats. The number of beats produced per second will be

(1) 3  (2) 2  (3) 1  (4) 4

Sol: Ans [3]

All three source will be in phase once in one second. Therefore beat frequency is one

80. A motor cycle starts from rest and accelerates along a straight path at \(2\text{m/s}^2\). At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94\% of its value when the motor cycle was at rest? (Speed of sound = 330 ms\(^{-1}\))

(1) 98 m  (2) 147 m  (3) 196 m  (4) 49 m

Sol: Ans [1]

Let \(v_0\) is velocity of observer when he hears frequency 0.94 \(f\)

\[
0.94f = f \left(1 - \frac{v_0}{330}\right)
\]

solving, \(v_0 = 19.8\) m/sec

using \(v^2 = u^2 + 2as\)

\[(19.8)^2 = 0 + 2 \times 2 \times S\]

\[
S = 98 \text{ m}
\]

81.

The above is a plot of binding energy per nucleon \(E_b\), against the nuclear mass \(M\); \(A, B, C, D, E, F\) correspond to different nuclei. Consider four reactions:

(i) \(A + B \rightarrow C + \varepsilon\)  (ii) \(C \rightarrow A + B + \varepsilon\)  (iii) \(D + E \rightarrow F + \varepsilon\)  (iv) \(F \rightarrow D + E + \varepsilon\)

Where \(\varepsilon\) is the energy released? In which reactions is \(\varepsilon\) positive?

(1) (i) and (iii)  (2) (ii) and (iv)  (3) (ii) and (iii)  (4) (i) and (iv)

Sol: Ans [4]

In (i) and (iv), Binding/nucleon is more for product elements
82. A transparent solid cylindrical rod has a refractive index of \( \frac{2}{\sqrt{3}} \). It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.

The incident angle \( \theta \) for which the light ray grazes along the wall of the rod is:

\[
(1) \quad \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \quad (2) \quad \sin^{-1} \left( \frac{2}{\sqrt{3}} \right) \quad (3) \quad \sin^{-1} \left( \frac{1}{\sqrt{3}} \right) \quad (4) \quad \sin^{-1} \left( \frac{1}{2} \right)
\]

Sol: Ans [3]

Using Snell’s law at O

\[
1 \times \sin \theta = \frac{2}{\sqrt{3}} \times \sin (90 - \theta_c)
\]

\[
\Rightarrow \quad \sin \theta = \frac{2}{\sqrt{3}} \cos \theta_c \quad \text{...(i)}
\]

Also \( \sin \theta_c = \frac{1}{\mu} = \frac{\sqrt{3}}{2} \)

\[
\Rightarrow \quad \cos \theta_c = \frac{1}{2}
\]

from (i), \( \sin \theta = \frac{2}{\sqrt{3}} \times \frac{1}{2} \)

\[
\Rightarrow \quad \theta = \sin^{-1} \left( \frac{1}{\sqrt{3}} \right)
\]

83. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area \( A \) and wire 2 has cross-sectional area \( 3A \). If the length of wire 1 increases by \( \Delta x \) on applying force \( F \), how much force is needed to stretch wire 2 by the same amount?

\[
(1) \quad 4F \quad (2) \quad 6F \quad (3) \quad 9F \quad (4) \quad F
\]

Sol: Ans [3]

\[
\frac{A l_1}{A l_2} = 3
\]

\[
\Rightarrow \quad \frac{l_1}{l_2} = \frac{1}{3} \quad \text{...(i)}
\]

\[
\frac{F}{A} = Y \cdot \frac{\Delta x}{l_1}
\]

\[
\frac{F}{3A} = Y \cdot \frac{\Delta x}{l_2}
\]

\[
\frac{F}{A} = \frac{F}{3A} \cdot \frac{3}{1} \Rightarrow \frac{F}{3A} = \frac{F}{A} \cdot \frac{1}{3}
\]

\[
\frac{F}{A} = Y \cdot \frac{\Delta x}{l_1}
\]

\[
\frac{F}{3A} = Y \cdot \frac{\Delta x}{l_2}
\]
Dividing
\[ \frac{3F}{F'} = \frac{l_2}{l_1} = \frac{1}{3} \]
\[ F' = 9F \]

This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements:

84. **Statement-1:** For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

**Statement-2:** The net work done by a conservative force on an object moving along a closed loop is zero.

(1) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1
(2) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true
(4) Statement-1 is true, Statement-2 is false

**Sol:** Ans [1]

85. The logic circuit shown below has the input waveforms A and B as shown. Pick out the correct output waveform

Output is

(1) (2) (3) (4)

**Sol:** Ans [4]

\[ Y = \overline{A+B} = \overline{A}.\overline{B} \]
\[ = A. B \text{ (using Boolean Algebra)} \]

86. If \( x \), \( v \) and \( a \) denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period \( T \), then, which of the following does not change with time?

(1) \( aT/x \)  
(2) \( aT+2\pi v \)  
(3) \( aT/v \)  
(4) \( a^2T^2 + 4\pi^2v^2 \)
87. A thin uniform rod of length \( l \) and mass \( m \) is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is \( \omega \). Its centre of mass rises to a maximum height of

\[
(1) \quad \frac{1}{6} \frac{l \omega}{g} \\
(2) \quad \frac{1}{2} \frac{l^2 \omega^2}{g} \\
(3) \quad \frac{1}{6} \frac{l^2 \omega^2}{g} \\
(4) \quad \frac{1}{3} \frac{l^2 \omega^2}{g}
\]

Sol: Ans [3]

Using conservation of mechanical energy

\[
mgh = \frac{1}{2} \times \frac{ml^2}{3} \times \omega^2
\]

\[
\Rightarrow h = \frac{l^2 \omega^2}{6g}
\]

88. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance \( u \) and the image distance \( v \), from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be

\[
(1) \quad \left( \frac{f}{2}, \frac{f}{2} \right) \\
(2) \quad (f, f) \\
(3) \quad (4f, 4f) \\
(4) \quad (2f, 2f)
\]

Sol: Ans [4]

At point of intersection of straight line with curve, \(|v| = |u|\)

It happens when \(|v| = |u| = 2f\)

89. A p-n junction (D) shown in the figure can act as a rectifier.

An alternating current source (V) is connected in the circuit.

The current \( I \) in the resistor \( R \) can be shown by:

\[
(1) \\
(2) \\
(3) \\
(4)
\]
90. Let \( P(r) = \frac{Q}{\pi R^4} r \) be the charge density distribution for a solid sphere of radius \( R \) and total charge \( Q \). For a point \( p \) inside the sphere at distance \( r_1 \) from the centre of the sphere, the magnitude of electric field is

\[ \begin{align*}
(1) & \quad \frac{Q}{4\pi \varepsilon_0 r_1^2} \\
(2) & \quad \frac{Qr_1^2}{4\pi \varepsilon_0 R^4} \\
(3) & \quad \frac{Qr_1^2}{3\pi \varepsilon_0 R^4} \\
(4) & \quad 0
\end{align*} \]

Sol: Ans [2]

Charge from \( r = 0 \) to \( r = r_1 \)

\[ q = \int_0^{r_1} \frac{Q}{\pi R^4} r \times 4\pi r^2 dr \]

\[ = \frac{Q}{\pi R^4} \times 4\pi \left( \frac{r_1^4}{4} \right) \]

\[ \Rightarrow q = \frac{Q}{R^4} r_1^4 \]

\[ E = \frac{1}{4\pi \varepsilon_0} \times \frac{Q}{R^4} r_1^4 = \frac{Qr_1^2}{4\pi \varepsilon_0 R^4} \]