MRF

PAPER - 1: MATHEMATICS , PHYSICS & CHEMISTRY

Test Booklet Code

D

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with *Blue/Black Ball point Pen. Use of pencil is strictly prohibited.*
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90 questions**. The maximum marks are **360.**
5. There are ***three*** parts in the question paper A, B, C consisting of **Mathematics, Physics,** and **Chemistry** having 30 questions in each part of equal weightage. Each questions is allotted **4 (four)** marks for correct response.
6. *Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing particulars / marking responses on ***Side-1*** and ***Side-2*** of the Answer Sheet use ***Blue / Black Ball Point*** provided by the Board.
9. No candidate is allowed to carry any textual material, printed or written, bits of Papers, pager, mobile phone, any electronic device, etc. except and Admit Card inside the examination room / hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **3** pages (Pages **20-23**) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. ***However, the candidates are allowed to take away this Test Booklet with them***.
12. The CODE for this Booklet is **D**. Make sure that the CODE printed on **Side-2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray marks on the Answer Sheet.**

Name of the Candidate (in Capital Letters): ....

Roll Number : ....

Examination Centre Number: ....

Name of Examination Centre (In Capital letters) ....

Candidate's Signature: .... Invigilator's Signature:: ....

PART A – MATHEMATICS

1. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is:

(1) $\frac{9}{2}$ (2) 6

(3) $\frac{7}{2}$ (4) 4

Solution (1):

$y^2 = 6x$

$\Rightarrow 2y\frac{dy}{dx} = 6 \Rightarrow \frac{dy}{dx} = \frac{3}{y}$

$9x^2 + by^2 = 16$

$\Rightarrow 18x + 2by\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{9x}{by}$

$m_1m_2 = -1$

$\Rightarrow \frac{3}{y}.-\frac{9x}{by} = -1$

$\Rightarrow 27x = by^2 = 6bx$

$\Rightarrow b = \frac{9}{2}$

2. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b} = 24$, then $|\vec{u}|^2$ is equal to:

(1) 84 (2) 336

(3) 315 (4) 256

Solution (2):

Let $\vec{u} = l\vec{a} + m\vec{b}$

$\vec{u}.\vec{a} = 0 \Rightarrow l|a|^2 + m\vec{a}.\vec{b} = 0$

$\vec{u}.\vec{b} = 24 \Rightarrow l\vec{a}.\vec{b} + m|b|^2 = 24$

$|a|^2 = 14, |b|^2 = 2, \vec{a}.\vec{b} = 2$

so $14l + 2m = 0$ and $2l + 2m = 24$

$\Rightarrow l = -2, m = 14$

so $\vec{u} = 14\vec{b} - 2\vec{a}$

$\Rightarrow |\vec{u}|^2 = 196|b|^2 + 4|a|^2 - 56\vec{a}.\vec{b} = 336$

3. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then

$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$

(1) does not exist (in R)

(2) is equal to 0

(3) is equal to 15

(4) is equal to 120

Solution (4):

$L = \lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$

$= \lim_{x\to 0^+} x\left[\left(\frac{1}{x} - \left\{\frac{1}{x}\right\}\right) + \left(\frac{2}{x} - \left\{\frac{2}{x}\right\}\right) + \ldots + \left(\frac{15}{x} - \left\{\frac{15}{x}\right\}\right)\right]$

$= \lim_{x\to 0^+} x\left[\left(\frac{1}{x} + \frac{2}{x} + \ldots \frac{15}{x}\right) - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots + \left\{\frac{15}{x}\right\}\right)\right]$

$= \lim_{x\to 0^+}\left[(1 + 2 + \ldots 15) - x\left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots \left\{\frac{15}{x}\right\}\right)\right]$

since $0 \le \{x\} < 1$

so $\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots \left\{\frac{15}{x}\right\}$ will be a finite number between 0 and 15.

$L = \lim_{x\to 0^+}[(1 + 2 + \ldots + 15) - 0] = 120$

4. Let L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is:

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{4\sqrt{2}}$

(3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

Solution (3):

Family of planes passing through L_1 is $(2x-2y+3z-2)+\lambda(x-y+z+1)=0$

$\Rightarrow x(2+\lambda)-y(2+\lambda)+(3+\lambda)-2+\lambda=0$ ..(i)

Family of planes passing through L_2 is

$(x+2y-z-3)+\mu(3x-y+2z-1)=0$

i.e.

$x(1+3\mu)+y(2-\mu)+z(-1+2\mu)-3-\mu=0$ ...(ii)

since (i) and (ii) are same planes,

$$\frac{2+\lambda}{1+3\mu}=-\frac{(2+\lambda)}{2-\mu}$$

$\Rightarrow (2+\lambda)(2-\mu+1+3\mu)=0$

$\Rightarrow \mu=-\frac{3}{2}\ (\lambda\neq-2)$

so required plane is $-7x+7y-8z=3$

Its distance from origin is $\frac{3}{\sqrt{162}}=\frac{1}{3\sqrt{2}}$

5. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin^2 x}{1+2^x}dx$ is;

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{8}$

(3) $\frac{\pi}{2}$ (4) 4π

Solution (1):

$$I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx \quad ....(1)$$

$$I=\int_{-\pi/2}^{\pi/2}\frac{2^x\sin^2 x}{2^x+1}dx \quad ....(2)$$

$$(1)+(2)\Rightarrow 2I=\int_{-\pi/2}^{\pi/2}\frac{1-\cos 2x}{2}dx$$

$$I=\frac{\pi}{4}$$

6. Let $g(x)=\cos x^2$, $f(x)=\sqrt{x}$, and $\alpha, \beta(\alpha<\beta)$ be the roots of the quadratic equation $18x^2-9\pi x+\pi^2=0$. Then the area (in sq. units) bounded by the curve $y=(gof)(x)$ and the lines $x=\alpha$, $x=\beta$ and $y=0$, is

(1) $\frac{1}{2}(\sqrt{2}-1)$ (2) $\frac{1}{2}(\sqrt{3}-1)$

(3) $\frac{1}{2}(\sqrt{3}+1)$ (4) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

Solution (2):

$(gof)(x)=g(f(x))=\cos x$

$18x^2-9\pi x+\pi^2=0$

$\Rightarrow (6x-\pi)(3x-\pi)=0$

$\Rightarrow x=\frac{\pi}{6},\frac{\pi}{3}$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\cos x=\sin x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}=\frac{\sqrt{3}-1}{2}$$

7. If sum of all the solutions of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0,\pi]$ is $k\pi$, then k is equal to:

(1) $\frac{20}{9}$ (2) $\frac{2}{3}$

(3) $\frac{13}{9}$ (4) $\frac{8}{9}$

Solution (3):

$$4\cos x\left[\cos\frac{\pi}{3}+\cos 2x-1\right]=1$$

$$4\cos x\left(\cos 2x-\frac{1}{2}\right)=1$$

$2\cos x(2\cos 2x-1)=1$

$2\cos x[(2\cos^2 x-1)-1]=1$

$2\cos x[4\cos^2 x-3]=1$

$2[4\cos 3x-3\cos x]=1$

$\cos 3x=\frac{1}{2}$

$\cos 3x = \cos\frac{\pi}{3}, \cos\left(2\pi - \frac{\pi}{3}\right),$

$\cos\left(2\pi + \frac{\pi}{3}\right)$

$3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

$k\pi = \frac{13\pi}{9}$

$k = \frac{13}{9}$

8. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is:

(1) $2\sqrt{2}$ (2) 3

(3) –3 (4) $-2\sqrt{2}$

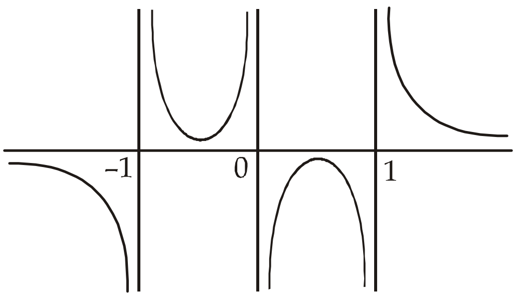
Solution (1):

$$h(x) = \frac{x^4 + 1}{x(x-1)(x+1)}$$

$$h(x) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}}$$

$$= \frac{\left(x - \frac{1}{x}\right)^2 + 2}{\left(x - \frac{1}{x}\right)}$$

$$= \left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)} \geq 2\sqrt{2}$$



[$x \in (-1, 0)$, then $x - (1/x) > 0$]

9. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

is equal to:

(1) $\frac{-1}{1 + \cot^3 x} + C$

(2) $\frac{1}{3(1 + \tan^3 x)} + C$

(3) $\frac{-1}{3(1 + \tan^3 x)} + C$

(4) $\frac{1}{1 + \cot^3 x} + C$

(where C is a constant of integration)

Solution (3):

$$\int \frac{\tan^2 x \sec^6 x}{(\tan^5 x + \tan^2 x + \tan^3 x + 1)^2} dx$$

$$= \int \frac{\tan^2 x \sec^6 x}{(1 + \tan^2 x)^2 (1 + \tan^3 x)^2} dx$$

$$= \int \frac{\tan^2 x \sec^2 x \, dx}{(1 + \tan^3 x)^2}$$

$$= -\frac{1}{3(1 + \tan^3 x)} + C$$

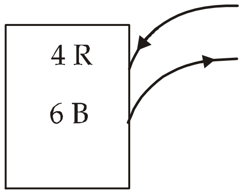
10. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(1) $\frac{3}{4}$ (2) $\frac{3}{10}$

(3) $\frac{2}{5}$ (4) $\frac{1}{5}$

Solution (3):

P(req) = BR + RR



$= \frac{6}{10}\times\frac{4}{12}+\frac{4}{10}\times\frac{6}{12}$

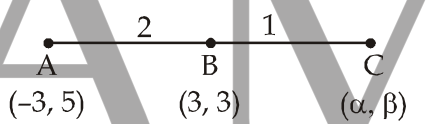
$= \frac{24+24}{10\times 12}=\frac{2}{5}$

11. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

(1) $\frac{3\sqrt{5}}{2}$ (2) $\sqrt{10}$

(3) $2\sqrt{10}$ (4) $3\sqrt{\frac{5}{2}}$

Solution (4):



C(6, 2)

$3=\frac{2\alpha-3}{\alpha+1}$

$2\alpha - 3 = 9$

$\alpha = 6$

$3 = \frac{2\beta+5}{2+1}$

$2\beta + 5 = 9$

$2\beta = 4$

$\beta = 2$

$r = \frac{1}{2}AC=\frac{1}{2}\sqrt{(6+3)^2+(5-2)^2}$

$= \frac{1}{2}\sqrt{81+9}$

$= \frac{\sqrt{90}}{2}=\frac{3\sqrt{10}}{2} = 3\sqrt{\frac{5}{2}}$

12. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$, then the value of c is:

(1) 95 (2) 195

(3) 185 (4) 85

Solution (1):

Equation of tangent

$x=\frac{y+7}{2}-6$

$2x = y + 7 - 12$

$2x = y - 5$

$2x - y + 5 = 0$

$p = r$

$\left|\frac{-16+6+5}{\sqrt{4+1}}\right| = \sqrt{64+36-c}$

$\left|\frac{5}{\sqrt{5}}\right| = \sqrt{100-c} \Rightarrow 100 - c = 5$

$c = 95$

13. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to:

(1) 2 (2) –1

(3) 0 (4) 1

Solution (4):

$\alpha = -\omega, \beta = -\omega^2$

$\beta^{101} + \beta^{107} = (-\omega)^{101} + (-\omega^2)^{107}$

$=-[\omega^{101} + \omega^{214}]$

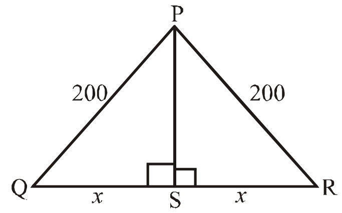
$= -[\omega + \omega^2] = 1$

14. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in *m*) is:

(1) $50\sqrt{2}$ (2) 100

(3) 50 (4) $100\sqrt{3}$

Solution (2):



$PS = \sqrt{40000 - x^2}$

Let height of tower = h

$\frac{h}{x} = \tan 30^\circ$

$\Rightarrow \quad x = \sqrt{3}\, h$

$\frac{h}{PS} = \tan 45^\circ$

$h = PS$

$\Rightarrow \quad h^2 = PS^2$

$h^2 = 40000 - 3h^2$

$h^2 = 10000$

$h = 100\ m$

15. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is:

(1) 3 (2) 9

(3) 4 (4) 2

Solution (4):

$\sum_{i=1}^{9}(x_i - 5) = 9$ **,** $\Sigma(x_i - 5)^2 = 45$

$\sigma = \sqrt{\frac{1}{9}\sum_{i=1}^{9}(x_i - \overline{x})^2}$

$\sum_{i=1}^{n} x_i - 5 \times 9 = 9$

$\Sigma x_i = 54$

$\overline{x} = \frac{54}{9} = 6$

$\sigma^2 = \frac{1}{9}\sum_{i=1}^{9}(x_i - 6)^2$

$= \frac{1}{9}\sum_{i=1}^{9}(x_i - 5 - 1)^2$

$= \frac{1}{9}\sum_{i=1}^{9}\left[(x_i - 5)^2 + 1 - 2(x_i - 5)\right]$

$= \frac{1}{9}[45 + 9 - 2 \times 9]$

$\sigma^2 = \frac{1}{9}[45 - 9] = \frac{36}{9} = 4$

$\sigma = 2$

16. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$, $(x > 1)$ is:

(1) 2 (2) –1

(3) 0 (4) 1

Solution (1):

$\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5$

$= 2x^5 + 20x^3(x^3 - 1) + 10x(x^3 - 1)^2$

Sum of coefficients of odd degree terms

= 2

17. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3), then the area (in sq. units) of ΔPTQ is:

(1) $36\sqrt{5}$ (2) $45\sqrt{5}$

(3) $54\sqrt{3}$ (4) $60\sqrt{3}$

Solution (2):

$\frac{x^2}{9} - \frac{y^2}{36} = 1$

$y = mx \pm \sqrt{9m^2 - 36}$ passes through (0, 3)

$\Rightarrow \quad m = \pm\sqrt{5}$

So, tangents are $y = \pm\sqrt{5}x + 3$

Solving these with the hyperbola,

$P \equiv \left(-3\sqrt{5}, -12\right)$ and $Q = \left(3\sqrt{5}, -12\right)$

So area = $\frac{1}{2}bh = 45\sqrt{5}$

18. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is:

(1) at least 750 but less than 1000

(2) at least 1000

(3) less than 500

(4) at least 500 but less than 750

Solution (2):

$^6C_4 \times {}^3C_1 \times 4!$

$= \frac{6\times5}{2}\times3\times4\times3\times2$ = 1080

19. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to:

(1) 30 (2) –10

(3) 10 (4) –30

Solution (3):

$$\begin{bmatrix}1 & k & 3\\ 3 & k & -2\\ 2 & 4 & -3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\begin{vmatrix}1 & k & 3\\ 3 & k & -2\\ 2 & 4 & -3\end{vmatrix}=0$$

$\Rightarrow$ $(-3k + 8) - k(-9 + 4) + 3(12 - 2k) = 0$

$-3k + 8 + 5k + 36 - 6k = 0$

$\Rightarrow$ $-4k + 44 = 0 \Rightarrow k = 11$

$x + 11y = -3z$

$3x + 11y = 2z$

$-2x = -5z$

$x = \frac{5}{2}z$

$11y = -3z - \frac{5}{2}z$

$y = \frac{-11}{11\times2}z \Rightarrow y = \frac{-z}{2}$

$\frac{xz}{y^2} = \frac{5\times4}{2} = 10$

20. If $\begin{vmatrix}x-4 & 2x & 2x\\ 2x & x-4 & 2x\\ 2x & 2x & x-4\end{vmatrix} = (A + Bx)(x - A)^2$,

then the ordered pair (A, B) is equal to

(1) (4, 5) (2) (–4, –5)

(3) (–4, 3) (4) (–4, 5)

Solution (4):

$$\begin{vmatrix}x-4 & 2x & 2x\\ 2x & x-4 & 2x\\ 2x & 2x & x-4\end{vmatrix} = (A + Bx)(x - A)^2$$

$C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$

$$= \begin{vmatrix}-x-4 & 0 & 2x\\ 0 & -x-4 & 2x\\ x+4 & x+4 & x-4\end{vmatrix}$$

$$= (x+4)^2\begin{vmatrix}-1 & 0 & 2x\\ 0 & -1 & 2x\\ 1 & 1 & x-4\end{vmatrix}$$

$R_1 \to R_1 + R_3$ = $(x+4)^2\begin{vmatrix}0 & 1 & 3x-4\\ 0 & -1 & 2x\\ 1 & 1 & x-4\end{vmatrix}$

$= (x + 4)^2(2x + 3x - 4)$

$= (x + 4)^2 (5x - 4)$

$\Rightarrow$ $A = -4, B = 5$ (–4, 5)

21. Two sets A and B are as under :

$A = \{(a, b) \in \mathbf{R} \times \mathbf{R} : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;

$B = \{(a, b) \in \mathbf{R} \times \mathbf{R} : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$. Then

(1) neither $A \subset B$ nor $B \subset A$

(2) $B \subset A$

(3) $A \subset B$

(4) $A \cap B = \phi$ (an empty set)

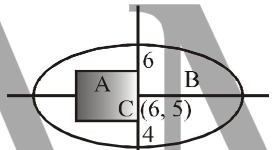
Solution (3):

$|a - 5| < 1 \Rightarrow a \in (4, 6)$

$|b - 5| < 1 \Rightarrow b \in (4, 6)$

$$\frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \leq 1$$

This is inside of an ellipse centred at (6, 5).



From the figure, $A \subset B$

22. Tangent and normal are drawn at $P(16, 16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is

(1) $\frac{4}{3}$ (2) $\frac{1}{2}$

(3) 2 (4) 3

Solution (3):

Tangent at (16, 16) is

$16y = 8(x + 16) \Rightarrow 2y = x + 16$

So, $A \equiv (-16, 0)$

Normal at (16, 16) is

$y = mx - 2am - am^3$

$m = -2$

$\Rightarrow y = -2x + 48$

So, $B \equiv (24, 0)$

So, $C \equiv (4, 0)$

Slope of $PC = \frac{4}{3}$ and Slope of $PB = -2$

$\Rightarrow \tan\theta = 2$

23. Let $S = \{t \in \mathbf{R} : f(x) = |x - \pi| \cdot (e^{|x|} - 1) \sin|x|$ is not differentiable at $t\}$. Then the set S is equal to

(1) $\{0, \pi\}$

(2) ϕ (an empty set)

(3) $\{0\}$

(4) $\{\pi\}$

Solution (2):

At $x = 0$, LHD =

$$\lim_{h\to 0} \frac{|-h - x|(e^{|-h|} - 1)\sin|-h| - 0}{-h}$$

$$= \lim_{h\to 0} |h + \pi|(e^h - 1)\left(\frac{\sinh}{h}\right) = 0$$

RHD = $\lim_{h\to 0} \frac{|h - \pi|(e^{|h|} - 1)\sin|h| - 0}{h} = 0$

LHD = RHD

At $x = \pi$, LHD =

$$\lim_{h\to 0} \frac{|\pi - h - \pi|(e^{|\pi - h|} - 1)\sin|\pi - h| - 0}{-h}$$

$$= \lim_{h\to 0} \frac{|h|(e^{\pi - h} - 1)\sinh}{h} = 0$$

RHD = $\lim_{h\to 0} \frac{|\pi + h - \pi| e^{(\pi + h - 1)} \sin h - 0}{h} = 0$

LHD = RHD

$\Rightarrow S = \phi$

24. The Boolean expression : $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to

(1) $\sim q$ (2) $\sim p$

(3) p (4) q

Solution (2):

(~*P* ^ ~*q*) *V* (~*p* ^ *q*)

~ *p* ^ (~*q* $\vee$ *q*) = ~*p* ^ *t* = ~*p*

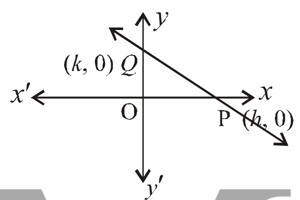
25. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points *P* and *Q*. If *O* is the origin and the rectangle *OPRQ* is completed, then the locus of *R* is

(1) $3x + 2y = 6xy$ (2) $3x + 2y = 6$

(3) $2x + 3y = xy$ (4) $3x + 2y = xy$

Solution (4):

$\frac{x}{h}+\frac{y}{k}=1$



$\frac{2}{h}+\frac{3}{k}=1$

$2k + 3h = hk$

$3x + 2y = xy$

26. Let *A* be the sum of the first 20 terms and *B* be the sum of the first 40 terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$

If $B - 2A = 100\lambda$, then λ is equal to

(1) 496 (2) 232

(3) 248 (4) 464

Solution (3):

$$A=\sum_{r=1}^{10}(2r-1)^2+8\sum_{r=1}^{10}r^2 = 4410$$

$$B=\sum_{r=1}^{20}(2r-1)^2+8\sum_{r=1}^{20}r^2 = 33620$$

$\therefore$ $B - 2A = 24800 = 100\lambda$ (given)

$\Rightarrow$ $\lambda = 248$

27. Let $y = y(x)$ be the solution of the differential equation $\sin x\frac{dy}{dx}+y\cos x=4x, x\in(0,\pi)$. If $y\left(\frac{\pi}{2}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $-\frac{4}{9}\pi^2$ (2) $\frac{4}{9\sqrt{3}}\pi^2$

(3) $\frac{-8}{9\sqrt{3}}\pi^2$ (4) $-\frac{8}{9}\pi^2$

Solution (4):

$$\frac{dy}{dx}+y(\cos x)=4x\operatorname{cosec} x$$

$\therefore$ I.F. = $e^{\int\cot x dx}=e^{\ln\sin x}=\sin x$

$\therefore$ $y\times\sin x=\int 4x\operatorname{cosec} x\times\sin x dx+C$

$\Rightarrow$ $y(x) = 2x^2 \operatorname{cosec} x + C \operatorname{cosec} x$

$\Rightarrow$ $y\left(\frac{\pi}{2}\right)=2\left(\frac{\pi^2}{4}\right)\times1+C \Rightarrow C=\frac{-\pi^2}{2}$

$\therefore$ $y(x)=2x^2\operatorname{cosec} x-\frac{\pi^2}{2}\operatorname{cosec} x$

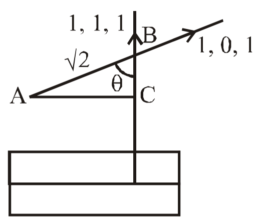
$\therefore$ $y\left(\frac{\pi}{6}\right)=\frac{-8}{9}\pi^2$

28. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

Solution (1):



$\cos\theta=\sqrt{\frac{2}{3}}$, $AB=\sqrt{1+0+1}=\sqrt{2}$

$$\sin\theta=\frac{AC}{AB}$$

$$AC=\sqrt{2}\sin\theta=\sqrt{2}\times\sqrt{1-\frac{2}{3}}=\sqrt{\frac{2}{3}}$$

29. Let $S = \{x \in \mathbf{R} : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S

(1) contains exactly four elements

(2) is an empty set

(3) contains exactly one element

(4) contains exactly two elements

Solution (4):

Case I

When $\sqrt{x} \geq 3 \Rightarrow x \geq 9$ (as $x \geq 0$)

$2\sqrt{x} - 6 + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 4\sqrt{x} = 0 \Rightarrow x = 0, 16$

Case II

When $\sqrt{x} < 3 \Rightarrow x < 9$

$\Rightarrow -2\sqrt{x} + 6 + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow (\sqrt{x} - 2)(\sqrt{x} - 6) = 0$

$\Rightarrow x = 4, 36$

$\Rightarrow x = 4, 16$ is the only solution

30. Let $a_1, a_2, a_3, \ldots a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140\,m$, then m is equal to

(1) 33 (2) 66

(3) 68 (4) 34

Solution (4):

$\sum_{k=0}^{12} a_{4k+1} = 416 \Rightarrow a + 24d = 32$ ...(i)

$\because a_9 + a_{43} = 66 \Rightarrow a + 25d = 33$ ...(ii)

From (i) & (ii) $a = 8, d = 1$

$\therefore a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$

$8^2 + 9^2 + 10^2 + \ldots + 24^2 = 140m$

$\Rightarrow 4760 = 140m \Rightarrow m = 34$

PART B – PHYSICS

ALL THE GRAPHS/DIAGRAMS GIVEN ARE SCHEMATIC AND NOT DRAWN TO SCALE

31. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities $+\sigma, -\sigma$ and $+\sigma$ respectively. The potential of shell B is

(1) $\frac{\sigma}{\varepsilon_o}\left[\frac{b^2 - c^2}{c} + a\right]$

(2) $\frac{\sigma}{\varepsilon_o}\left[\frac{a^2 - b^2}{a} + c\right]$

(3) $\frac{\sigma}{\varepsilon_o}\left[\frac{a^2 - b^2}{b} + c\right]$

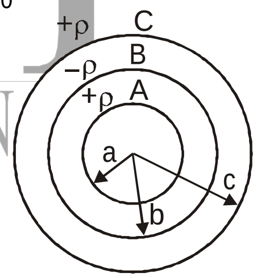
(4) $\frac{\sigma}{\varepsilon_o}\left[\frac{b^2 - c^2}{b} + a\right]$

Solution (3)

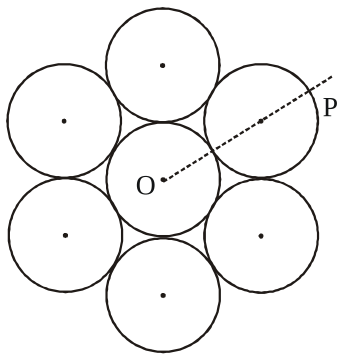
$$V_B = \frac{\sigma a^2}{\varepsilon_0 b} - \frac{\sigma b}{\varepsilon_0} + \frac{\sigma c}{\varepsilon_0}$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$



32. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is



(1) $\frac{181}{2}MR^2$

(2) $\frac{19}{2}MR^2$

(3) $\frac{55}{2}MR^2$

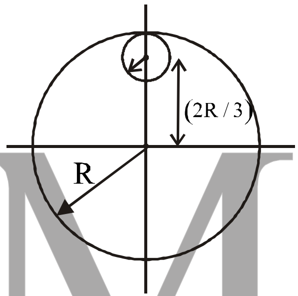
(4) $\frac{73}{2}MR^2$

Solution (1)

$$I_0 = 7 \times \frac{1}{2}MR^2 + 6 \times M \times 4R^2 = \frac{55}{2}MR^2$$

$$I = \frac{55}{2}MR^2 + 7M \times 9R^2 = \frac{181}{2}MR^2$$

33. From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is



(1) $\frac{37}{9}MR^2$

(2) $4MR^2$

(3) $\frac{40}{9}MR^2$

(4) $10MR^2$

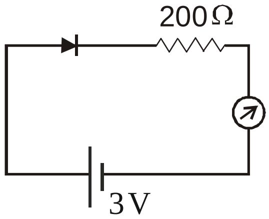
Solution (2)

$$I = \frac{1}{2} \times 9M \times R^2$$

$$-\left(\frac{1}{2}M\frac{R^2}{9} + M \times \frac{4R^2}{9}\right)$$

$I = 4MR^2$

34. The reading of the ammeter for a silicon diode in the given circuit is :



(1) 13.5 mA (2) 0

(3) 15 mA (4) 11.5 mA

Solution (4)

Knee voltage for silicon diode is 0.7 V

$I = \frac{3 - 0.7}{200}$ = 11.5 mA

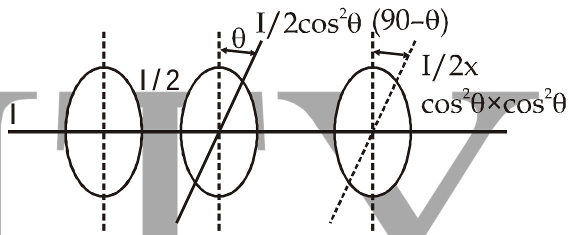
35. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is :

(1) 60° (2) 0°

(3) 30° (4) 45°

Solution (4)

Axis of A & B coincide



$$\frac{I}{2}\cos^4\theta = \frac{I}{8}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

36. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_o = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by

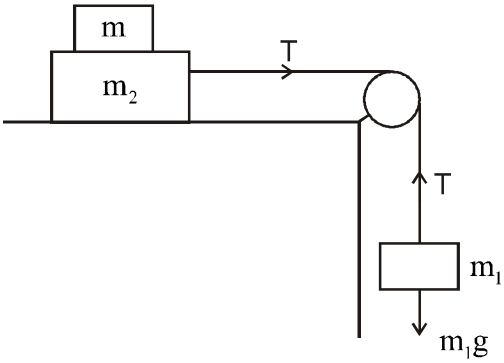
(1) $\frac{CR}{\omega_o}$ (2) $\frac{\omega_o L}{R}$

(3) $\frac{\omega_o R}{L}$ (4) $\frac{R}{(\omega_o C)}$

Solution (2)

$$Q = \frac{\omega_0 L}{R} \because Q = \frac{X_L}{R}$$

37. Two masses m_1 = 5 kg and m_2 = 10 kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is :



(1) 10.3 kg (2) 18.3 kg

(3) 27.3 kg (4) 43.3 kg

Solution (3)

$$m_1 g = \mu (m + m_2) g$$

5 = 0.15 (m + 10)

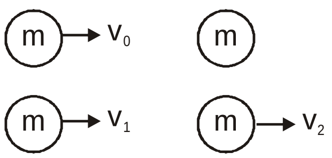
$$m = \frac{100}{3} - 10 = \frac{70}{3} = 23.3\text{ kg}$$

38. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(1) $\frac{v_0}{\sqrt{2}}$ (2) $\frac{v_0}{4}$

(3) $\sqrt{2}v_0$ (4) $\frac{v_0}{2}$

Solution (3)



$$mv_0 = mv_1 + mv_2 \quad \text{...(i)}$$

$$1.5 \times \frac{1}{2} mv_0^2 = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2$$

$$\frac{3}{2} v_0^2 = v_1^2 + v_2^2 \quad \text{...(ii)}$$

squaring equation (i) both side

$$v_1^2 + v_2^2 + 2v_1 v_2 = v_0^2$$

$$\therefore \quad 2v_1 v_2 = -\frac{v_0^2}{2}$$

$$v_1 v_2 = \frac{-v_0^2}{4}$$

$$v_1 - v_2 = \sqrt{(v_1 + v_2)^2 - 4v_1 v_2}$$

$$= \sqrt{v_0^2 + v_0^2} = \sqrt{2} v_0$$

39. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then :

(1) $T \propto R^{n/2}$

(2) $T \propto R^{3/2}$ for any n.

(3) $T \propto R^{\frac{n}{2}+1}$

(4) $T \propto R^{(n+1)/2}$

Solution (4)

$$F = \frac{k}{R^n}$$

$$F = \frac{mv^2}{R}$$

$$\frac{k}{R^n} = \frac{mv^2}{R} \Rightarrow v \propto \frac{1}{R^{\frac{n-1}{2}}}$$

$$T = \frac{2\pi R}{v}$$

$$\Rightarrow T \propto R^{1+\left(\frac{n-1}{2}\right)}$$

$$T \propto R^{\frac{n+1}{2}}$$

40. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10Ω. The internal resistances of the two batteries are 1Ω and 2Ω respectively. The voltage across the load lies between

(1) 11.7 V and 11.8 V

(2) 11.6 V and 11.7 V

(3) 11.5 V and 11.6 V

(4) 11.4 V and 11.5 V

Solution (3)

$$E_{eq} = \frac{24+13}{3} = \frac{37}{3} V$$

$$r_{eq} = \frac{2}{3}\Omega$$

$$V = \left(\frac{37/3}{10+2/3}\right) \times 10 = \frac{370}{32} V = 11.56 \text{ V}$$

41. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively

(1) 50, 0 (2) 50, 10

(3) $\frac{1000}{\sqrt{2}}, 10$ (4) $\frac{50}{\sqrt{2}}, 0$

Solution (3)

$P = V_{rms}\, I_{rms} \cos\phi$

$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \cos\frac{\pi}{4} = \frac{1000}{\sqrt{2}}$ W

Wattless current I = $I_{rms} \sin\phi$

$= \frac{20}{\sqrt{2}} \sin 45°$

= 10 A

42. An EM wave from air enters a medium. The electric fields are

$\vec{E}_1 = E_{01}\hat{x} \cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air and

$\vec{E}_2 = E_{02}\hat{x} \cos\left[k(2z - ct)\right]$ in medium,

where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic. If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$ (2) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$

(3) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$ (4) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

Solution (4)

$$v = \frac{1}{\sqrt{(\varepsilon_0\varepsilon_r)(\mu_0\mu_r)}}$$

$$\Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}} = \frac{1}{2}$$

$$\Rightarrow \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$$

43. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(1) 2×10^6 (2) 2×10^3

(3) 2×10^4 (4) 2×10^5

Solution (4)

$$n = \frac{\text{channel width}}{\text{band width required}}$$

$$= \frac{10\times10^9}{10\times5\times10^3} = 2\times10^5$$

44. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7\times10^3\ kg/m^3$ and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 7.5 kHz (2) 5 kHz

(3) 2.5 kHz (4) 10 kHz

Solution (2)

$$v=\sqrt{\frac{Y}{\rho}}=\sqrt{\frac{92.7}{2.7}\times10^6}$$

and $\frac{\lambda}{4}=\frac{\ell}{2}\Rightarrow\lambda=1.2\ m$

$$\nu=\frac{v}{\lambda}=4.88\times10^3\ Hz\approx5\ kHz$$

45. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon molecules at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively :

(1) (0, 1) (2) (0.89, 0.28)

(3) (0.28, 0.89) (4) (0, 0)

Solution (2)

For collision with deuterium

$$v_1=\left(\frac{m_1-m_2}{m_1+m_2}\right)u$$

$\Rightarrow$ for dentvium, $v_1=\left(\frac{2-1}{2+1}\right)u=\frac{u}{3}$

loss of KE = $\left(1-\left(\frac{1}{3}\right)^2\right)$ = 0.89

For collision with carbon atom,

for carbon $v_1=\left(\frac{12-1}{12+1}\right)u=\left(\frac{11}{13}\right)u$

loss of KE = $\left(1-\left(\frac{11}{13}\right)^2\right)$ = 0.28

46. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(1) 6% (2) 2.5%

(3) 3.5% (4) 4.5%

Solution (4)

$$\rho=\frac{m}{a^3}$$

$$\Rightarrow\ \frac{\Delta\rho}{\rho}=\frac{\Delta m}{m}+\frac{3\Delta a}{a}$$

$= (1.5+3\times1)\%=4.5\%$

47. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 195 K (b) 2.7 kJ

(2) (a) 189 K (b) 2.7 kJ

(3) (a) 195 K (b) –2.7 kJ

(4) (a) 189 K (b) –2.7 kJ

Solution (4)

$$T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$$

$$\Rightarrow\ \frac{T_2}{300}=\left(\frac{1}{2}\right)^{\left(\frac{5}{3}-1\right)}$$

$\Rightarrow$ T_2 = 189 K

$$\Delta U=nC_V\ \Delta T=\frac{3}{2}nR\Delta T=-2.7\ kJ$$

48. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is

(1) $\frac{mg}{Ka}$ (2) $\frac{Ka}{mg}$

(3) $\frac{Ka}{3mg}$ (4) $\frac{mg}{3Ka}$

Solution (4)

$$K = \frac{\Delta P}{(-\Delta V / V)} \Rightarrow -\frac{\Delta V}{V} = \frac{mg}{Ka}$$

but $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{dV}{V} = \frac{3dr}{r} \Rightarrow \frac{-dr}{r} = \frac{mg}{3Ka}$$

49. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a deelectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be

(1) 0.9 n C (2) 1.2 n C

(3) 0.3 n C (4) 2.4 n C

Solution (2)

C = 90 × 5/3 = 150 pF

Q = CV = 150 × 20 = 3000 pF

$$q_{ind} = \left(1 - \frac{1}{K}\right)Q$$

= 1.2 nC

50. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is

(1) $\frac{1}{\sqrt{2}}$ (2) 2

(3) $\sqrt{3}$ (4) $\sqrt{2}$

Solution (4)

$$m_1 = I\pi r_1^2$$

$$m_2 = 2m_1$$

$$\Rightarrow I\pi r_2^2 = 2I\pi r_1^2$$

$$\Rightarrow r_2 = \sqrt{2} r_1$$

$$B = \frac{\mu_0 I}{2r}$$

$$\Rightarrow \frac{B_1}{B_2} = \frac{r_2}{r_1} = \sqrt{2}$$

51. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let A_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n^2 \approx \lambda$ (2) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

(3) $\Lambda_n \approx A + B\lambda_n$ (4) $\Lambda_n^2 \approx A + B\lambda_n^2$

Solution (2)

$$n = \frac{\lambda_n}{\lambda_g}$$

when electron falls from n = n to n = 1

then $\frac{hc}{\lambda} = E_0\left(\frac{1}{1^2} - \frac{1}{n^2}\right) = E_0\left(1 - \left(\frac{\lambda_g}{\lambda_n}\right)^2\right)$

$$\Rightarrow \quad \frac{\lambda}{hc} \propto \left[1 - \left(\frac{\lambda_g}{\lambda_n}\right)^2\right]^{-1}$$

$$\Rightarrow \quad \lambda \propto 1 + \frac{\lambda_g^2}{\lambda_n^2}$$

$$\Rightarrow \quad A_n \approx A + \frac{B}{\lambda_n^2}$$

52. the mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m / s, then the pressure on the wall is nearly

(1) 4.70×10^2 N / m^2
(2) 2.35×10^3 N / m^2
(3) 4.70×10^3 N / m^2
(4) 2.35×10^2 N / m^2

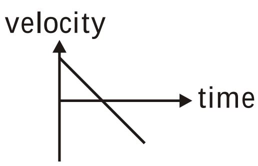
Solution (3)

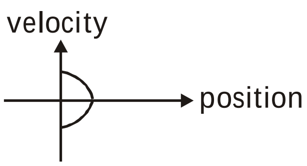
$$\text{Pressure} = \frac{2nmv\cos\theta}{A}$$

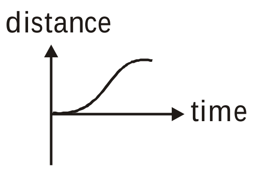
$$= \frac{3.32 \times 10^{-27} \times 10^{23} \times 10^3}{2 \times 10^{-4}} \times \frac{1}{\sqrt{2}} \times 2$$

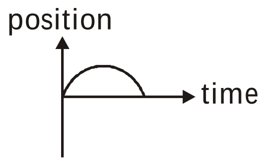
= 2.3×10^3 N / m^2

53. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

(1) 

(2) 

(3) 

(4) 

Solution (3)

Initially speed is decreasing with time

54. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is

(1) $r_e < r_\alpha < r_p$ (2) $r_e > r_p = r_\alpha$
(3) $r_e < r_P = r_\alpha$ (4) $r_e < r_p < r_\alpha$

Solution (3)

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

$$\Rightarrow \quad r \propto \frac{\sqrt{m}}{q}$$

$$\Rightarrow \quad r_\alpha = r_p > r_e$$

55. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchnaging the resistances ?

(1) 910 Ω (2) 990 Ω
(3) 505 Ω (4) 550 Ω

Solution (4)

$R_1 + R_2 = 1000$

$$\frac{R_1}{R_2} = \frac{11}{9}$$

$$\Rightarrow \quad R_1 = 550\,\Omega$$

56. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell si connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 2.5 Ω (2) 1 Ω

(3) 1.5 Ω (4) 2 Ω

Solution (3)

$$r = R\left(\frac{\ell_1}{\ell_2}-1\right) = 5\left(\frac{52}{40}-1\right)$$

$$\Rightarrow \quad r = 1.5\,\Omega$$

57. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is

(1) ν_L / 25 (2) 25 ν_L

(3) 16 ν_L (4) ν_L / 16

Solution (1)

$$\nu_L = Rc\left(1-\frac{1}{\infty}\right) = Rc$$

$$\nu_P = Rc\left(\frac{1}{5^2}-\frac{1}{\infty}\right) = \frac{Rc}{25}$$

$$\Rightarrow \quad \frac{\nu_P}{\nu_L} = \frac{1}{25}$$

58. The angular width of the central maximum in a single slit diffraction patter is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance ?

(i.e. distance between the centres of each slit.)

(1) 100 μm (2) 25 μm

(3) 50 μm (4) 75 μm

Solution (2)

$$\sin\,\theta = \frac{2\lambda}{b} \Rightarrow \lambda = \frac{1}{2}\times 10^{-6}\ \text{m}$$

$$w = \frac{D}{d}\lambda$$

$$\Rightarrow \quad 10^{-2} = \frac{1/2\times 10^{-6}\times 0.5}{d}$$

$$\Rightarrow \quad d = 25\ \mu\text{m}$$

59. A particle is moving in a circular path of a radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is

(1) $-\frac{3}{2}\frac{k}{a^2}$ (2) $-\frac{k}{4a^2}$

(3) $\frac{k}{2a^2}$ (4) Zero

Solution (4)

$$F = \frac{-dU}{dr} = -\frac{2K}{2r^3} = -\frac{K}{r^3}$$

$$\Rightarrow \quad \frac{mv^2}{r} = \frac{k}{r^3}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}\frac{K}{r^2}$$

T.E. = U + KE

$$= \quad -\frac{K}{2r^2}+\frac{K}{2r^2} = \text{Zero}$$

60. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver = 108 and Avagadro number = 6.02 × 10^{23} gm mol^{-1})

(1) 5.5 N/m (2) 6.4 N/m

(3) 7.1 N/m (4) 2.2 N/m

Solution (3)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\Rightarrow \quad 10^{-12} = 2\pi\sqrt{\frac{108\times 10^{-3}}{6.02\times 10^{23}k}}$$

$$\Rightarrow \quad k = 7.07\ \text{N/m}$$

PART C – CHEMISTRY

61. For 1 molal aqueous solution of the following commpounds, which one will show the highest freezing point?

(1) $[Co(H_2O)_3Cl_3].3H_2O$
(2) $[Co(H_2O)_6]Cl_3$
(3) $[Co(H_2O)_5Cl]Cl_2.H_2O$
(4) $[Co(H_2O)_4Cl_2Cl.2H_2O$

Solution (1):

More is the value of *i* (Van't Hoff factor) higher is the depression in freezing point, hence lesser freezing point.

62. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively:

(1) H_2O and $(H_2O + OH^-)$
(2) $(H_2O + O_2)$ and H_2O
(3) $(H_2O + O_2)$ and $(H_2O + OH^-)$
(4) H_2O and $(H_2O + O_2)$

Solution (4):

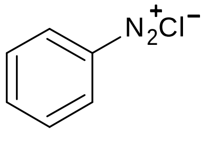
$$[Fe(CN)_6]^{4-} + H_2O_2 \longrightarrow [Fe(CN)_6]^{3-} + H_2O$$

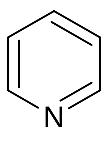
H_2O_2 acts as oxidising agent, hence itself gets reduced into H_2O.

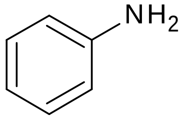
$$[Fe(CN)_6]^{3-} + H_2O_2 \longrightarrow [Fe(CN)_6]^{4-} + H_2O + O_2$$

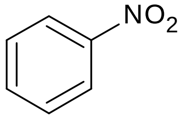
H_2O_2 acts as reducing agent, hence itself gets oxidised into O_2.

63. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

(1) 

(2) 

(3) 

(4) 

Solution (3):

Kjeldahl's method is given by those compound which have ammonia or ammonia derivative.

64. Glucose on prolonged heating with HI gives:

(1) 6-iodohexanal (2) n-Hexane
(3) 1-Hexene (4) Hexanoic acid

Solution (2):

With HI; all aldehyde, ketones and alcoholic group reduced into alkane.

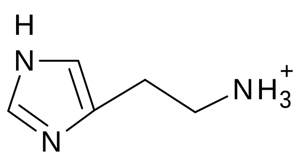
65. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

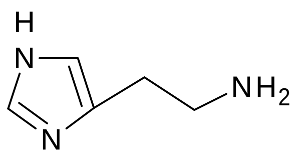
	Base	Acid	End point
(1)	Strong	Strong	Pink to colourless
(2)	Weak	Strong	Colourless to pink
(3)	Strong	Strong	Pinkish red to yellow
(4)	Weak	Strong	Yellow to pinkish red

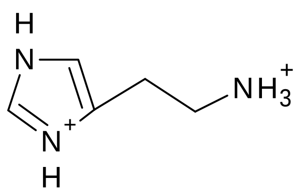
Solution (4):

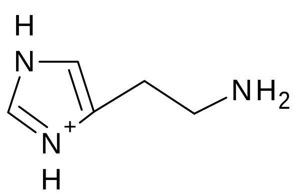
pH range for methyl orange indicator is 3.1 to 4.4, hence in acidic medium it changes yellow to orange.

66. The predominant form of histamine present in human blood is (pK_a, Histidine = 6.0)

(1) 

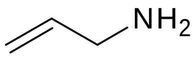
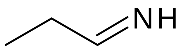
(2) 

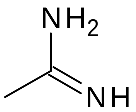
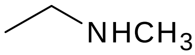
(3) 

(4) 

Solution (1): Factual

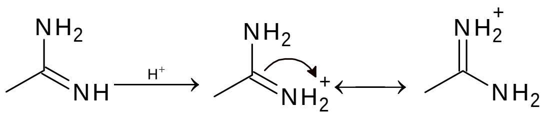
67. The increasing order of basicity of the following compounds is:

(a)  (b) 

(c)  (d) 

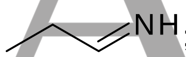
(1) (d) < (b) < (a) < (c)
(2) (a) < (b) < (c) < (d)
(3) (b) < (a) < (c) < (d)
(4) (b) < (a) < (d) < (c)

Solution (4):

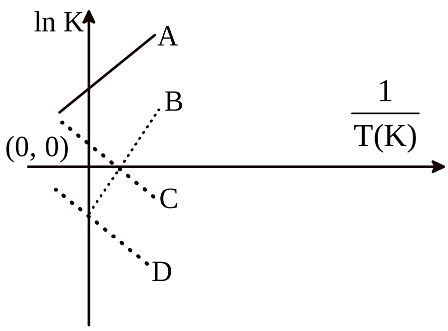


Conjugate acid is resonance stabilized hence it has maximum basic strength.

; + I inductive effect of CH_3 and C_2H_5 increases its basic strength.

; N is sp^2 hybridised hence its basic strength is less.

68. Which of the following lines correctly show the temperature dependence of equilibrium constnat, K, for an exothermic reaction?



(1) A and D (2) A and B
(3) B and C (4) C and D

Solution (2):

Van't Hoff equation

$$\ln K = \frac{-\Delta H}{RT} + \frac{\Delta S}{R}$$

For exothermic reaction ΔH =–ve hence slope becomes positive.

69. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane?

(Atomic weight of B = 10.8 u)

(1) 1.6 hours (2) 6.4 hours
(3) 0.8 hours (4) 3.2 hours

Solution (4):

$$H_2O \longrightarrow H_2 + \frac{1}{2}O_2$$

$$B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$$

27.66 g of B_2H_6 required 3 mole of O_2 to completely burn B_2H_6.

Hence moles of H_2O electrolysed is 6.

w = ZIt

$$6 = \frac{1 \times 100 \times t}{2 \times 96500}$$

t = 3.2 hours

70. Consider the following reaction and statements:

$$[Co(NH_3)_4Br_2]^+ + Br^- \longrightarrow [Co(NH_3)_3Br_3] + NH_3$$

(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer.

(II) Two isomers are produced if the reactant complex ion is a *trans*-isomer.

(III) Only one isomer is produced if the reactant complex ion is a trans-isomer.

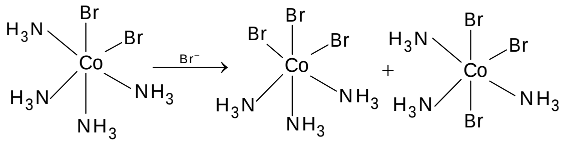
(IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.

The correct statements are:

(1) (II) and (IV) (2) (I) and (II)
(3) (I) and (III) (4) (III) and (IV)

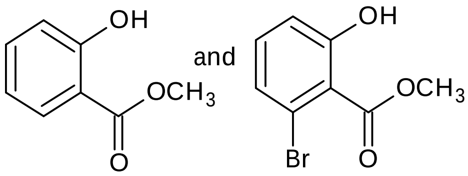
Solution (3):

If reactant complex ion is *cis.*



71. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively.

(1)



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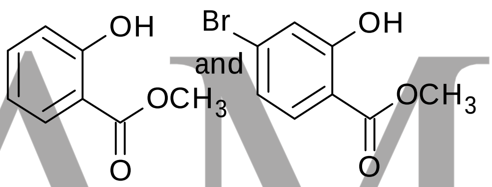

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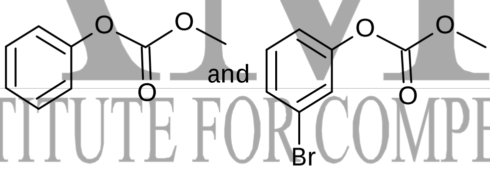

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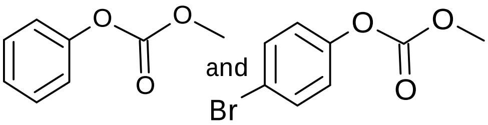


Solution (4):

72. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just beings to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+}?

(1) 1.0×10^{-10} M (2) 5×10^{-9} M

(3) 2×10^{-9} M (4) 1.1×10^{-9} M

Solution (4):

$$Ba^{2+} + Na_2SO_4 \longrightarrow BaSO_4$$

50 × 1 M 50 milli mole

(M × V)

Molarity of $[SO_4^{2-}] = \frac{5 \times 10^{-2} \times 1000}{500} = 0.1M$

K_{sp} of $Ba^{2+}SO_4^{2-}$

$1 \times 10^{-10} = [Ba^{2+}]\,[0.1]$

$[Ba^{2+}] = 1 \times 10^{-9}$ M

$M_1V_1 = M_2V_2$

$1 \times 10^{-9} \times 500 = M_2 \times 450$

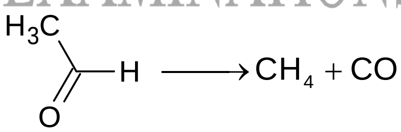
$M_2 = 1.1 \times 10^{-9}$

73. At 518°C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted with 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is:

(1) 0 (2) 2

(3) 3 (4) 1

Solution (2):



t = 0 363 torr

Rate	Initial conc.
1.00	344.85
0.5	243.21

Let order of reaction be p

$$\text{Rate} = k\left[CH_3 - \underset{\underset{O}{\|}}{C} - H\right]^p$$

$1.00 = k[344.85]^p$

$0.5 = k[243.21]^p$

p = 2

74. The combustion of benzene (l) gives CO_2(g) and H_2O(l). Given that heat of combustion of benzene at constant volume is – 3263.9 kJ mol^{-1} at 25°C, heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be

(1) – 3267.6 (2) 4152.6

(3) – 452.46 (4) 3260

Solution (1):

$$C_6H_{6(l)} + \frac{15}{2}O_{2(g)} \longrightarrow 6CO_{2(g)} + 3H_2O_{(l)}$$

$\Delta H = \Delta U + \Delta ngRT$

$\Delta H = -3263.9 + (-1.5) \times 8.314 \times 10^{-3} \times 298$

$\Delta H = -3267.6$

75. The ratio of mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is

(1) $C_2H_4O_3$ (2) $C_3H_6O_3$

(3) C_2H_4O (4) $C_3H_4O_2$

Solution (1):

$$\frac{12x}{y} = \frac{6}{1}$$

$2x = y$ for $C_xH_yO_z$

$$C_xH_y(g) + \left(x + \frac{y}{4}\right)O_{2(g)} \longrightarrow xCO_{2(g)} + \frac{y}{2}H_2O_{(l)}$$

no. of oxygen atom in $C_xH_yO_z$ = z

no. of oxygen atom required for C_xH_y combustion is $\left(x + \frac{y}{4}\right) \times 2 = \left(2x + \frac{y}{2}\right)$

So, $z = \frac{1}{2}\left(2x + \frac{y}{2}\right)$

$$z = x + \frac{y}{4}$$

$$z = x + \frac{2x}{4} = \frac{3x}{2}$$

$$x : 2x : \frac{3x}{2}$$

2x : 4x : 3x

2 : 4 : 3

Hence $C_2H_4O_3$

76. The trans-alkenes are formed by the reduction of alkynes with

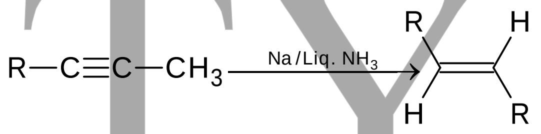
(1) Sn-HCl

(2) H_2-Pd/C, $BaSO_4$

(3) $NaBH_4$

(4) Na/liq. NH_3

Solution (4):



Trans-alkene

77. Which of the following are Lewis acids?

(1) BCl_3 and $AlCl_3$

(2) PH_3 and BCl_3

(3) $AlCl_3$ and $SiCl_4$

(4) PH_3 and $SiCl_4$

Solution (1):

Lewis acid is a species which accept pair of electrons.

BCl_3 and $AlCl_3$ are electron deficient species and accept pair of electrons.

78. When metal M is treated with NaOH, a white gelatinous precipitate X is obtained, which is soluble in excess of NaOH. Compound X when heated strongly gives an oxide which is used in chromatograph as an adsorbent. The metal M is

(1) Fe (2) Zn

(3) Ca (4) Al

Solution (4):

$$2Al + 2NaOH + 6H_2O \longrightarrow 2Na[Al(OH)_4] + 3H_2$$

Al_2O_3 is used in chromatography.

79. According to molecular orbital theory, which of the following will not be a viable molecule?

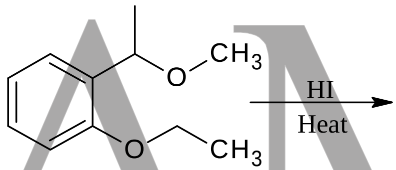
(1) H_2^{2-} (2) He_2^{2+}

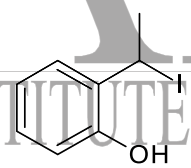
(3) He_2^{+} (4) H_2^{-}

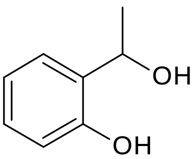
Solution (1):

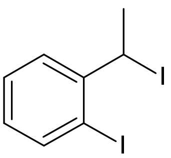
Bond order of $H_2^{2-} = 0$

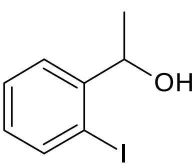
80. The major product formed in the following reaction is



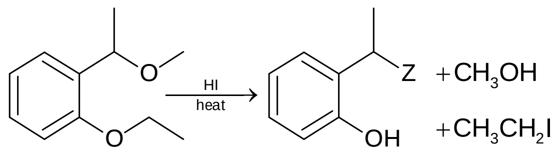
(1) 

(2) 

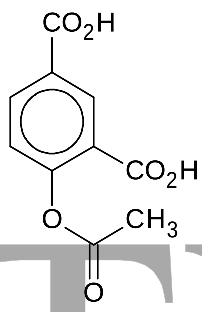
(3) 

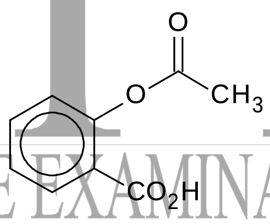
(4) 

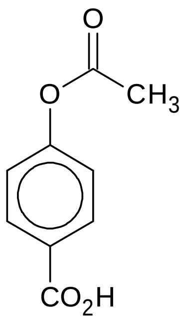
Solution (1):

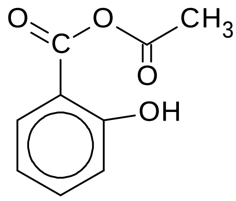


81. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the kanor product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces

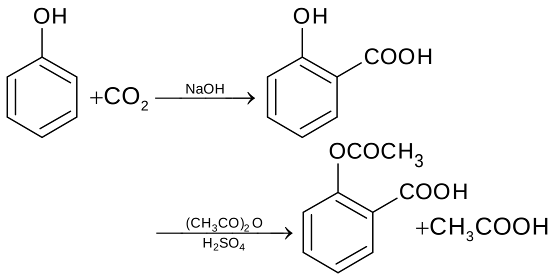
(1) 

(2) 

(3) 

(4) 

Solution (2):



82. Which of the following compounds contain(s) no covalent bonds (s)?

KCl, PH_3, O_2, B_2H_6, H_2SO_4

(1) KCl, B_2H_6

(2) HCl, B_2H_6, PH_3

(3) KCl, H_2SO_4

(4) KCl

Solution (4):

KCl is an ionic compound.

83. Which type of defect has the presence of cations in the interstitial sites?

(1) Metal deficiency defect

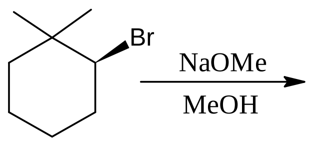
(2) Schottky defect

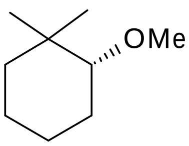
(3) Vacancy defect

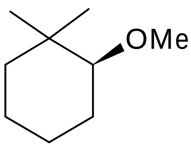
(4) Frenkel defect

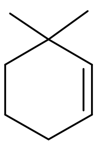
Solution (4):

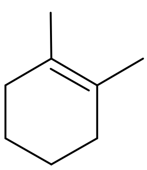
84. The major product of the following reaction

is 

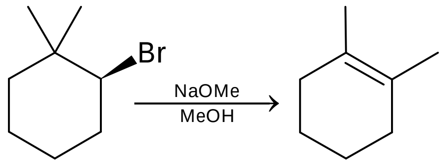
(1) 

(2) 

(3) 

(4) 

Solution (4):



85. The compound that does not produce nitrogen gas by the thermal decomposition is

(1) $(NH_4)_2SO_4$ (2) $Ba(N_3)_2$

(3) $(NH_4)_2Cr_2O_7$ (4) NH_4NO_2

Solution (3):

$$(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$$

$$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$$

$$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$$

$$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$$

86. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constants for the formation of HS from H_2S is 1.0×10^{-7} and that of S^{2-} from HS ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is

(1) 5×10^{-19} (2) 5×10^{-8}

(3) 3×10^{-20} (4) 6×10^{-21}

Solution (4):

In presence of HCl; ionisation of H_2S is also suppressed.

(H^+) conc. given by HCl is 2.0×10^{-1} M and hence (H^+) from H_2S may be neglected.

$$H_2S \rightleftharpoons H^+ + HS^- \quad ; \quad K_1 = 1.0 \times 10^{-7}$$

$$H_2S \rightleftharpoons H^+ + S^{2-} \quad ; \quad K_2 = 1.2 \times 10^{-13}$$

$$H_2S \rightleftharpoons 2H^+ + S^{2-} \; ; \quad K_3 = K_1 \times K_2 = 1.2 \times 10^{-20}$$

$$K_3 = \frac{[H^+]^2[S^{2-}]}{[H_2S]}$$

$$1.2\times10^{-20} = \frac{(2.0\times10^{-1})^2(S^{2-})}{0.1}$$

$$= \frac{1.2\times10^{-20}\times10^{-1}}{4\times10^{-2}} = [S^{2-}]$$

$$= \frac{12\times10^{-21}\times10^{-1}\times10^{2}}{4} = [S^{2-}]$$

$= 3.0 \times 10^{-20}$ M $= [S^{2-}]$

87. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$ respectively are

(1) +3, 0 and +4 (2) +3, +4 and +6

(3) +3, +2 and +4 (4) +3, 0 and +6

Solution (4):

$[Cr(H_2O)_6]Cl_3$: $[Cr(C_6H_6)_2]$

O.S. of Cr = + 3 O.S. of Cr = 0

Since C_6H_6 = 0

$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$

Oxidation state Cr = + 6

Since CN^- = – 1

O = –2

O_2^{2-} = –2

88. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2Ca(OH)_2]$ to

(1) $[3\{Ca(OH)_2\}.CaF_2]$

(2) $[CaF_2]$

(3) $[3(CaF_2).Ca(OH)_2]$

(4) $[3Ca_3(PO_4)_2.CaF_2]$

Solution (4):

$[3Ca_3(PO_4)_2.CaF_2$

89. Which of the following salts is the most basic in aqueous solution?

(1) $Pb(CH_3COO)_2$ (2) $Al(CN)_3$

(3) CH_3COOK (4) $FeCl_3$

Solution (3):

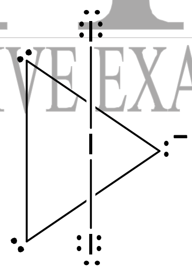
CH_3COOK is the salt of strong base (KOH) and weak acid (CH_3COOH)

90. Total number of lone pair of electrons in I_3^- ion is

(1) 12 (2) 3

(3) 6 (4) 9

Solution (4):



Total no. of lone pair of electrons = 9

Read the following instructions carefully

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet (***Side-1***) with ***Blue/Black Ball Point Pen***.

2. For writing / marking particulars on **Side-2** of the Answer Sheet, use ***Blue/Black Ball Point Pen only.***

3. The candidates should not write their Roll Numbers anywhere else (except in the specified space) on the Test Booklet / Answer Sheet.

4. Out of four options given for each question, only one option is the correct answer.

5. For each ***incorrect response, one-fourth*** (1 / 4) of the total marks allotted to the question would be deduced from the total score. ***No deduction*** from the total score, however, will be made if ***no response*** is indicated for an item in the Answer Sheet.

6. Handle the Test Booklet and Answer Sheet with care, *as under no circumstances (except for discrepancy in Test Booklet Code and Answer Sheet Code), another set will be provided*.

7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations / writing work are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in 3 pages (Pages 20-23) at the end of the booklet.

8. On completion of the test, the candidates must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

9. Each candidate must show on demand his / her Admit Card to the Invigilator.

10. No candidate, without special permission of the Superintendent or Invigilator, should leave his / her seat.

11. The candidates should not leave the Examination Hall without handing over their Answer Sheet tot he Invigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed the Attendance Sheet a second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. **The candidates are also required to put their left hand THUMB impression in the space provided in the Attendance Sheet.**

12. Use of Electronic / Manual Calculator and any electronic Item like mobile phone, pager etc. is prohibited.

13. The candidates are governed by all Rules and Regulations of the JAB / Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the JAB / Board.

14. **Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination hall/room.**

ANS KEY-2018 -JEE MAINS

PART A – PHYSICS

1.	(1)	2.	(2)	3.	(4)	4.	(3)	5.	(1)
6.	(2)	7.	(3)	8.	(1)	9.	(3)	10.	(3)
11.	(4)	12.	(1)	13.	(4)	14.	(2)	15.	(4)
16.	(1)	17.	(2)	18.	(2)	19.	(3)	20.	(4)
21.	(3)	22.	(3)	23.	(2)	24.	(2)	25.	(4)
26.	(3)	27.	(4)	28.	(1)	29.	(4)	30.	(4)

PART B – MATHEMATICS

31.	(3)	32.	(1)	33.	(2)	34.	(4)	35.	(4)
36.	(2)	37.	(3)	38.	(3)	39.	(4)	40.	(3)
41.	(3)	42.	(4)	43.	(4)	44.	(2)	45.	(2)
46.	(4)	47.	(4)	48.	(4)	49.	(2)	50.	(4)
51.	(2)	52.	(3)	53.	(3)	54.	(3)	55.	(4)
56.	(3)	57.	(1)	58.	(2)	59.	(4)	60.	(3)

PART C – CHEMISTRY

61.	(1)	62.	(4)	63.	(3)	64.	(2)	65.	(4)
66.	(1)	67.	(4)	68.	(2)	69.	(4)	70.	(3)
71.	(4)	72.	(4)	73.	(2)	74.	(1)	75.	(1)
76.	(4)	77.	(1)	78.	(4)	79.	(1)	80.	(1)
81.	(2)	82.	(4)	83.	(4)	84.	(4)	85.	(3)
86.	(4)	87.	(4)	88.	(4)	89.	(3)	90.	(4)