MATHEMATICS

SECTION- I

STRAIGHT OBJECTIVE TYPE

This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choice (A), (B), (C) and (D), out of which ONLY-ONE is correct

1. Let

$$I = \int \frac{e^x}{e^{4x} + e^{2x} + 1}\, dx, \; J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1}\, dx$$

Then, for an arbitrary constant C, the value of $J - I$ equals

(A) $\frac{1}{2}\log\left(\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1}\right) + C$

(B) $\frac{1}{2}\log\left(\frac{e^{2x} + e^{x} + 1}{e^{2x} - e^{x} + 1}\right) + C$

(C) $\frac{1}{2}\log\left(\frac{e^{2x} - e^{x} + 1}{e^{2x} + e^{x} + 1}\right) + C$

(D) $\frac{1}{2}\log\left(\frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1}\right) + C$

Sol. Ans [C]

$$J - I = \int\left(\frac{e^{3x}}{e^{4x} + e^{2x} + 1} - \frac{e^x}{e^{4x} + e^{2x} + 1}\right)dx = \int \frac{e^x(e^{2x} - 1)}{e^{4x} + e^{2x} + 1}dx = \int \frac{(t^2 - 1)}{t^4 + t^2 + 1}dt$$

$$= \int \frac{1 - 1/t^2}{t^2 + \frac{1}{t^2} + 1}dt = \int \frac{dz}{z^2 - 1}, \text{ where } z = t + \frac{1}{t}$$

$$= \frac{1}{2}\log\left|\frac{z-1}{z+1}\right| + c = \frac{1}{2}\log\left|\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1}\right| + c$$

2. Let $g(x) = \log f(x)$ where $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x + 1) = x f(x)$. Then, for $N = 1, 2, 3, ...$

$$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$$

(A) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N-1)^2}\right\}$

(B) $4\left\{1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N-1)^2}\right\}$

(C) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N+1)^2}\right\}$

(D) $4\left\{1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N+1)^2}\right\}$

Sol: Ans [A]

$g(x) = \log f(x)$

$g(x+1) = \log f(x+1) = \log(x f(x)) = \log x + g(x)$

$g''(x+1) - g''(x) = -\dfrac{1}{x^2}$

Putting $x = \dfrac{1}{2}, \dfrac{3}{2}, ..., \dfrac{2N-1}{2}$ and adding, we get

$$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) = -4\left(1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N-1)^2}\right)$$

3. Let two non-collinear unit vector $\hat{a}$ and $\hat{b}$ form an acute angle. A point *P* moves so that at any time *t* the position vector $\overrightarrow{OP}$ (where *O* is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When *P* is farthest from origin *O*, let *M* be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the vector along $\overrightarrow{OP}$. Then,

(A) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + \hat{a}\cdot\hat{b})^{1/2}$

(B) $\hat{u} = \dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 + \hat{a}\cdot\hat{b})^{1/2}$

(C) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + 2\hat{a}\cdot\hat{b})^{1/2}$

(D) $\hat{u} = \dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 + 2\hat{a}\cdot\hat{b})^{1/2}$

Sol: Ans [A]

$|\overrightarrow{OP}|^2 = ((\hat{a}\cos t + \hat{b}\sin t)(\hat{a}\cos t + \hat{b}\sin t)$

$= 1 + \hat{a}\cdot\hat{b}\sin(2t)$

For maximum value needed $\sin 2t = 1$

$\Rightarrow$ $M = (1 + \hat{a}\cdot\hat{b})^{1/2}$

$\Rightarrow$ for maximum value $t = \pi/4$

$\Rightarrow$ $\overrightarrow{OP} = \dfrac{\hat{a}+\hat{b}}{\sqrt{2}}$

$\Rightarrow$ $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$

4. Let the function $g : (-\infty, \infty) \to \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \dfrac{\pi}{2}$. Then, *g* is

(A) even and is strictly increasing in $(0, \infty)$

(B) odd and is strictly decreasing in $(-\infty, -\infty)$

(C) odd and is strictly increasing in $(-\infty, \infty)$

(D) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

Sol. Ans [D]

$g(u) = 2\tan^{-1}(e^u) - \pi/2$

$g'(u) = \dfrac{2e^u}{1+e^{2u}} > 0$ $\Rightarrow$ [$g(u)$ is strictly increasing in $-\infty, \infty$]

Clearly it is neither even nor odd.

5. Consider a branch of the hyperbola

$x^2 - 2y^2 - 2\sqrt{2}\,x - 4\sqrt{2}\,y - 6 = 0$

with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(A) $1 - \sqrt{\dfrac{2}{3}}$ (B) $\sqrt{\dfrac{3}{2}} - 1$ (C) $1 + \sqrt{\dfrac{2}{3}}$ (D) $\sqrt{\dfrac{3}{2}} + 1$

Sol. Ans [B]

$x^2 - 2y^2 - 2\sqrt{2x} - 4\sqrt{2}y - 6 = 0$

$\left(x - \sqrt{2}\right)^2 - 2\left(y + \sqrt{2}\right)^2 = 4$

$\Rightarrow$ $a^2 = 4$, $b^2 = 2$

$e = \dfrac{\sqrt{6}}{2}$

For point A, $\left(x - \sqrt{2}\right) = a,\ \left(y + \sqrt{2}\right) = 0$

So, $A\left(2 + \sqrt{2}, -\sqrt{2}\right)$

For point B, $\left(ae, \dfrac{b^2}{a}\right)$

So, $B\left(\sqrt{6} + \sqrt{2}, 1 - \sqrt{2}\right)$

Similarly focus $C\left(\sqrt{2} + \sqrt{6}, -\sqrt{2}\right)$

Hence, area of the triangle is $\left(\sqrt{\dfrac{3}{2}} - 1\right)$sq. units

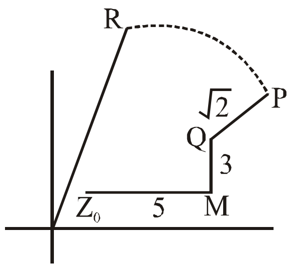
6. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\mathbf{i} + \mathbf{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by

(A) $6 + 7i$ (B) $-7 + 6i$ (C) $7 + 6i$ (D) $-6 + 7i$

Sol. Ans [D]

Z_0 is $1 + 2\,i$ and M is (6, 2), Q is (6, 5)

Coordinates of P are $\dfrac{x-6}{\cos 45^\circ} = \dfrac{y-5}{\sin 45^\circ} = \sqrt{2}$



$\Rightarrow$ $x = 7$, $y = 6$

Let $R = (x + iy)$

So, $\dfrac{x + iy - 0}{7 + 6i - 0} = e^{i\pi/2}$

$\Rightarrow$ $x + iy = -6 + 7i$

7. The area of the region between the curves $y = \sqrt{\dfrac{1+\sin x}{\cos x}}$ and $y = \sqrt{\dfrac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \dfrac{\pi}{4}$ is

(A) $\displaystyle\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$ (B) $\displaystyle\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(C) $\displaystyle\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$ (D) $\displaystyle\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

Sol: Ans [B]

$$y = \sqrt{\frac{1+\sin x}{\cos x}} = \sqrt{\frac{1+\cos((\pi/2)-x)}{\sin((\pi/2)-x)}} = \sqrt{\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)}$$

$$y = \sqrt{\frac{1-\sin x}{\cos x}} = \sqrt{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)}$$

Now $0 \le x \le \pi/4$

$$\frac{\pi}{4} \ge \frac{\pi}{4} - \frac{x}{2} \ge \frac{\pi}{8}$$

$$\tan\left(\frac{\pi}{4}-\frac{x}{2}\right) \le 1 \;\Rightarrow\; \sqrt{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)} \le 1$$

$$\Rightarrow\; \sqrt{\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)} \ge 1$$

So $\sqrt{\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)} \ge \sqrt{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}$

So Area $= \int_0^{\pi/4}\left(\sqrt{\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)} - \sqrt{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)dx$

$$= 2\int_0^{\pi/4} \frac{\sin(x/2)}{\cos(x/2)\sqrt{1-\tan^2(x/2)}}\,dx$$

Putting $\tan(x/2) = t \Rightarrow dt = (1/2)\sec^2(x/2)\,dx$

$$= \int_0^{\tan\frac{\pi}{8}} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$$

$(\tan\frac{\pi}{8} = \sqrt{2} - 1)$

$$= \int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$$

8. An experiment has 10 equally likely outcomes. Let *A* and *B* be two non-empty events of the experiment. If *A* consists of 4 outcomes, the number of outcomes that *B* must have so that *A* and *B* are independent, is

(A) 2, 4 or 8 (B) 3, 6 or 9 (C) 4 or 8 (D) 5 or 10

Sol. Ans [D]

No. of possible outcomes = 10

$n(A) = 4$

Let $n(B) = k$ and $n(A \cap B) = x$ where x lies between 1 and 4

Because *A* and *B* are independent, so $P(A \cap B) = P(A)\,.\,P(B)$

So, $\frac{x}{10} = \frac{4}{10}.\frac{K}{10}$

So, $K = \frac{5}{2}x$

x can be 2 and 4 only as K is an integer. So, $K = 5$ or 10.

9. Consider three points

$P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$,

where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then,

(A) P lies on the segment RQ

(B) Q lies on the line segment PR

(C) R lies on the segment QP

(D) P, Q, R are non-collinear

Sol: Ans [D]

$$\Delta = \begin{vmatrix} \cos(\beta-\alpha) & \sin\beta & 1 \\ -\sin(\beta-\alpha) & -\cos\beta & 1 \\ \cos(\beta-\alpha-\theta) & \sin(\beta-\theta) & 1 \end{vmatrix}$$

$$R_3 \to R_3 - (\cos\theta\, R_1 + \sin\theta\, R_2)$$

$$= \begin{vmatrix} \cos(\beta-\alpha) & \sin\beta & 1 \\ -\sin(\beta-\alpha) & -\cos\beta & 1 \\ 0 & 0 & 1-(\cos\theta+\sin\theta) \end{vmatrix}$$

$= -[1 - (\cos\theta + \sin\theta)] \cos(2\beta - \alpha)$

$2\beta < \pi/2$ and $\alpha > 0$

$\Rightarrow \quad (2\beta - \alpha) < \pi/2 \quad \Rightarrow \quad \cos(2\beta - \alpha) \neq 0$

Also since $0 < \theta < \pi/4 \Rightarrow \quad \cos\theta + \sin\theta > 1$

$\Rightarrow \quad \Delta \neq 0$

$\Rightarrow$ P, Q and R are not collinear.

SECTION- II

ASSERTION–REASON TYPE

This section contains 4 multiple choice questions numbered 10 to 13. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

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10. Suppose four distinct positive numbers, a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT-1: The number b_1, b_2, b_2, b_4 are neither in A.P. nor in G.P.

and

STATEMENT-2: The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Sol: Ans [C]

$a_1 = a$ $a_2 = ar$ $a_3 = ar^2$ $a_4 = ar^3$

$b_1 = a$ $b_2 = a(1 + r)$ $b_3 = a(1 + r + r^2)$ $b_4 = a(1 + r + r^2 + r^3)$

$\Rightarrow$ $b_1 = a$ $b_2 = \dfrac{a(1-r^2)}{1-r}$ $b_3 = \dfrac{a(1-r^3)}{1-r}$ $b_4 = \dfrac{a(1-r^4)}{1-r}$

$\Rightarrow$ $\dfrac{1}{b_1} = \dfrac{1}{a}$ $\dfrac{1}{b_2} = \dfrac{1-r}{a(1-r^2)}$ $\dfrac{1}{b^3} = \dfrac{1-r}{a(1-r^3)}$ $\dfrac{1}{b_4} = \dfrac{1-r}{a(1-r^4)}$

$\Rightarrow$ (D) is the correct answer.

11. Consider

$L_1 : 2x + 3y + p - 3 = 0$

$L_2 : 2x + 3y + p + 3 = 0,$

where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$.

STATEMENT-1: If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

and

STATEMENT-2: If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Sol: Ans [D]

Given circle is $x^2 + y^2 + 6x - 10y + 30 = 0$

$\Rightarrow$ $(x + 3)^2 + (y - 5)^2 = 2^2$ centre is $(-3, 5)$

$\Rightarrow$ If L_1 is chord then $\left|\dfrac{-6+15+p-3}{\sqrt{4+9}}\right| < 2$

$\Rightarrow$ $\left|\dfrac{p+6}{\sqrt{13}}\right| < 2$ $\Rightarrow$ $-2\sqrt{13} < p + 6 < 2\sqrt{13}$ $\Rightarrow$ $-2\sqrt{13} - 6 < p < 2\sqrt{13} - 6$

For L_2 to be diameter, $-6 + 15 + p + 3 = 0 \Rightarrow p = -12$

$\Rightarrow$ Statement 1 is false $\Rightarrow$ (D) is answer.

12. Let a solution $y = y(x)$ of the differential equation

$$x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$$

satisfy $y(2) = \dfrac{2}{\sqrt{3}}$.

STATEMENT-1: $y(x) = \sec\left(\sec^{-1} x - \frac{\pi}{6}\right)$

and

STATEMENT-2: $y(x)$ is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Sol: Ans [C]

$$\int \frac{dy}{y\sqrt{y^2 - 1}} = \int \frac{dx}{x\sqrt{x^2 - 1}}$$

$\Rightarrow$ $y = \sec(\sec^{-1} - \pi/6)$ $\Rightarrow$ sect 1 is true.

Again, $y = \frac{1}{\cos(\sec^{-1} x - \pi/6)} = \frac{1}{\cos(\sec^{-1} x)\cos\pi/6 + \sin(\sec^{-1} x)\sin\pi/6}$

$y = \dfrac{1}{\frac{\sqrt{3}}{2x} + \frac{\sqrt{x^2 - 1}}{2x}}$ $\Rightarrow$ (Sect. 2 is false)

13. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$.

STATEMENT-1: $(p^2 - q)(b^2 - ac) \geq 0$

and

STATEMENT-2: $b \neq pa$ or $c \neq qa$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Sol: Ans [B]

Since $a\alpha^2 + 2b\alpha + c = 0$ .....(1)

$\alpha^2 + 2p\alpha + q = 0$ .... (2)

$\Rightarrow \quad \frac{a}{1} \neq \frac{2b}{2p} \neq \frac{c}{q} \quad \Rightarrow \quad b \neq pa, \quad c \neq aq$

Again $(p^2 - q) \geq 0$ and $(b^2 - ac) > 0 \Rightarrow (p^2 - q)(b^2 - ac) \geq 0$

SECTION- III

LINKED COMPREHENSION TYPE

This section contains 2 Paragraphs $P_{14\text{-}16}$ and $P_{17\text{-}19}$. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

$P_{14\text{-}16}$: Paragraph for Question Nos. 14 to 16

Consider the function $f : (-\infty, \infty) \to (-\infty, \infty)$ defined by

$$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}, \; 0 < a < 2$$

14. Which of the following is true?

(A) $(2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0$

(B) $(2 - a)^2 f''(1) + (2 + a)^2 f''(-1) = 0$

(C) $f'(1)\, f'(-1) = (2 - a)^2$

(D) $f'(1)\, f'(-1) = -(2 + a)^2$

Sol: Ans [A]

$$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1} = 1 - \frac{2ax}{x^2 + ax + 1}$$

$$f'(x) = -2a \frac{[(x^2 + ax + 1) - x(2x + a)]}{(x^2 + ax + 1)^2}$$

$$= \frac{2a(x^2 - 1)}{(x^2 + ax + 1)^2}$$

$$f''(x) = \frac{4a[-x^3 + 3x + a]}{(x^2 + ax + 1)^3}$$

$$\Rightarrow \quad f''(1) = \frac{4a}{(2 + a)^2}, \qquad f''(-1) = \frac{-4a}{(2 - a)^2}$$

$$\Rightarrow \quad (2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0$$

15. Which of the following is true?

(A) $f(x)$ is decreasing on $(-1, 1)$ and has a local minimum at $x = 1$

(B) $f(x)$ is increasing on $(-1, 1)$ and has local maximum at $x = 1$

(C) $f(x)$ is increasing on (–1, 1) but has neither a local maximum nor a local minimum at $x = 1$

(D) $f(x)$ is decreasing on (–1, 1) but has neither a local maximum nor a local minimum at $x = 1$

Sol: Ans [A]

$$f'(x) = \frac{2a(x^2 - 1)}{(x^2 + ax + 1)^2}$$

$f'(x) < 0 \Rightarrow x^2 - 1 < 0 \Rightarrow x \in (-1, 1)$

So in (–1, 1), $f(x)$ is decreasing

$$f''(1) = \frac{4a}{(2 + a)^2} > 0 \text{ and } f'(1) = 0$$

So $x = 1$ is point of local minima.

16. Let $g(x) = \int_0^{e^x} \frac{f'(t)}{1 + t^2}\, dt$. Which of the following is true?

(A) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$

(B) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$

(C) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$

(D) $g'(x)$ does not change sign on $(-\infty, \infty)$

Sol: Ans [B]

$$g(x) = \int_0^{e^x} \frac{f'(t)}{1 + t^2}\, dt$$

$$\Rightarrow g'(x) = \frac{f'(e^x)}{1 + e^{2x}} \cdot e^x - 0$$ (by Leibnitz rule of differentiation)

Now $\frac{e^x}{1 + e^{2x}} > 0$

So $g'(x) > 0 \Rightarrow f'(e^x) > 0 \Rightarrow e^x > 1$

(since $f(x)$ is increasing in $(-\infty, -1) \cup (1, \infty)$)

$\Rightarrow x > 0$

So $g'(x)$ is positive in $(0, \infty)$

$g'(x) < 0 \Rightarrow f'(e^x) < 0 \Rightarrow 0 < e^x < 1$

(Since $f(x)$ is decreasing in (–1, 1))

$\Rightarrow x < 0$

So $g'(x)$ is negative in $(-\infty, 0)$

$P_{17\text{-}19}$: Paragraph for Question Nos. 17 to 19

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$$

$$L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

17. The unit vector perpendicular to both L_1 and L_2 is

(A) $\frac{-\mathbf{i}+7\mathbf{j}+7\mathbf{k}}{\sqrt{99}}$ (B) $\frac{-\mathbf{i}-7\mathbf{j}+5\mathbf{k}}{5\sqrt{3}}$ (C) $\frac{-\mathbf{i}+7\mathbf{j}+5\mathbf{k}}{5\sqrt{3}}$ (D) $\frac{7\mathbf{i}-7\mathbf{j}-\mathbf{k}}{\sqrt{99}}$

Sol: Ans [B]

dr's for line perpendicular to L_1 and L_2 given by (–1, –7, 5)

$\Rightarrow$ Unit vector = $\frac{-\mathbf{i}-7\mathbf{j}+5\mathbf{k}}{5\sqrt{3}}$

18. The shortest distance between L_1 and L_2 is

(A) 0 (B) $\frac{17}{\sqrt{3}}$ (C) $\frac{41}{5\sqrt{3}}$ (D) $\frac{17}{5\sqrt{3}}$

Sol: Ans [D]

Shortest distance between L_1 and L_2 be

$$= \left| (1+2)\frac{(-1)}{5\sqrt{3}} + (2-2)\frac{(-7)}{5\sqrt{3}} + (1+3)\frac{5}{5\sqrt{3}} \right| = \frac{17}{5\sqrt{3}}$$

19. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(A) $\frac{2}{\sqrt{75}}$ (B) $\frac{7}{\sqrt{75}}$ (C) $\frac{13}{\sqrt{75}}$ (D) $\frac{23}{\sqrt{75}}$

Sol: Ans [C]

Equation of plane will be

$(x + 1)(-1) + (y + 2)(-7) + (3 + 1)(-1) = 0$

$\Rightarrow$ $x + 7y - 52 + 10 = 0$

$\Rightarrow$ distance = $\left| \frac{1+7-5+10}{\sqrt{1+49+25}} \right| = \frac{13}{\sqrt{75}}$

SECTION- IV

MATRIX-MATCH TYPE

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statement (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II.

20. Consider the lines given by

$L_1 : x + 3y - 5 = 0$

$L_2 : 3x - ky - 1 = 0$

$L_3 : 5x + 2y - 12 = 0$

Match the Statements/Expressions in Column-I with the Statements/Expressions in Column-II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 given in the ORS.

Column I	Column II
(A) L_1, L_2, L_3 are concurrent, if	(p) $k = -9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q) $k = -6/5$
(C) L_1, L_2, L_3 form a triangle, if	(r) $k = 5/6$
(D) L_1, L_2, L_3 do not form a triangle, if	(s) $k = 5$

Sol: Ans [A-(s); B-(p),(q); C-(r); D-(p),(q),(s)]

(A) Lines L_1, L_2 and L_3 are concurrent

If $\begin{vmatrix} 1 & 3 & -5 \\ 3 & -k & -1 \\ 5 & 2 & -12 \end{vmatrix} = 0 \Rightarrow \quad k = 5$

(B) The lines have slopes $-\frac{1}{3}, -\frac{5}{2}, \frac{3}{k}$

So atleast two lines are parallel

if $\frac{3}{k} = -\frac{1}{3}$ or $\frac{3}{k} = -\frac{5}{2}$

$\Rightarrow \quad k = -9$ or $k = -\frac{6}{5}$

(C) Lines will form a triangle in all other cases except *A* and *B* i.e., for $k = \frac{5}{6}$

(D) Lines will not form a triangle in (A) and (B)

i.e., for $k = -9, -\frac{6}{5}, 5$

21. Consider all possible permutations of the letters of the word ENDEANOEL.

Match the Statements/Expressions in Column-I with the Statements/Expressions in Column-II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 given in the ORS.

Column I	Column II
(A) The number of permutations containing the word ENDEA is	(p) $5!$
(B) The number of permutations in which the letter E occurs in the first and the last positions is	(q) $2 \times 5!$
(C) The number of permutations in which none of the letters D, L, N occurs in the last five positions is	(r) $7 \times 5!$
(D) The number of permutations in which the letters A, E, O occur only in odd positions is	(s) $21 \times 5!$

Sol: Ans [A-(p); B-(s); C-(q); D-(q)]

(A) Considering ENDEA as one group, remaining letters are N, O, E, L

So no. of permutations = $5!$

(B) E occurs in Ist and last positions.

Remaining letters are N, N, D, A, O, E, L

No. of permutations = $\frac{7!}{2!} = \frac{7 \times 6}{2} \times 5! = 21 \times 5!$

(C) D, L, N should not occur in last five positions

$\Rightarrow$ D, L, N should occur in Ist four positions, but we have D, L, N, N

So ways of arranging D, L, N, N in Ist four positions = $\frac{4!}{2!} = 12$

Ways of arranging remaining E, E, A, O, E in last five positions = $\frac{5!}{3!} = 20$

Total no. of permutations = $12 \times 20 = 240 = 2 \times 5!$

(D) A, E, O occur in odd positions

No of odd positions = 5

and letters are E, E, E, A, O i.e., 5

Ways of arranging these 5 letters in 5 odd positions = $\frac{5!}{3!} = 20$

Remaining 4 letters D, L, N, N can be arranged in remaining 4 positions in $\frac{4!}{2!} = 12$ ways

Total no. of permutations = $20 \times 12 = 240 = 2 \times 5!$

22. Match the Statements/Expressions in Column-I with the Statements/Expressions in Column-II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 given in the ORS.

Column I	Column II
(A) The minimum value of $\frac{x^2+2x+4}{x+2}$ is	(p) 0
(B) Let A and B be 3 × 3 matrices of real numbers, where A is symmetric, B is skew-symmetric, and $(A+B)(A-B) = (A-B)(A+B)$. If $(AB)^t = (-1)^k AB$, where $(AB)^t$ s the transpose of the matrix AB, then the possible values of k are	(q) 1
(C) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	(r) 2
(D) If $\sin\theta = \cos\varphi$, then the possible values of $\frac{1}{\pi}\left(\theta \pm \varphi - \frac{\pi}{2}\right)$ are	(s) 3

Sol: Ans [A-(r); B-(q),(s); C-(r),(s); D-(p),(r)]

(A)
$$\frac{x^2+2x+4}{x+2} = \frac{x^2+2x}{x+2} + \frac{4}{x+2}$$
$$= x + \frac{4}{x+2}$$
$$= x + 2 + \frac{4}{x+2} - 2$$

Now,
$$\frac{x+2+4/(x+2)}{2} \geq \sqrt{(x+2)\frac{4}{x+2}}$$
$$\Rightarrow \quad x + 2 + \frac{4}{x+2} \geq 4$$
$$\Rightarrow \quad x + 2 + \frac{4}{x+2} - 2 \geq 2$$

(B) $(A+B)(A-B) = (A-B)(A+B)$

$\Rightarrow \quad A^2 + BA - AB - B^2 = A^2 - B^2 + AB - BA$

$\Rightarrow \quad 2BA = 2AB$

$\Rightarrow \quad BA = AB \qquad$ ...(i)

Now $(AB)^t = B^tA^t = (-B)A \quad$ (since A is symmetric B is skew-symmetric)

$= -BA$

$= -AB \qquad$ (Using (i))

$\Rightarrow \quad k = \text{odd} \quad \Rightarrow \quad k = 1, 3$

(C) $a = \log_3 \log_3 2$

$\Rightarrow \quad 3^a = \log_3 2$

$\Rightarrow \quad 3^{-a} = \dfrac{1}{3^a} = \dfrac{1}{\log_3 2} = \log_2 3$

So $\quad 2^{-k+3^{-a}} = 2^{-k+\log_2 3} = 2^{-k} \cdot 2^{\log_2 3} = 3 \cdot 2^{-k}$

So $\quad 1 < 3 \cdot 2^{-k} < 2$

$\Rightarrow \quad \dfrac{1}{3} < 2^{-k} < \dfrac{2}{3}$

$\Rightarrow \quad \log_2 \dfrac{1}{3} < -k < \log_2 \dfrac{2}{3}$

$\Rightarrow \quad -\log_2 3 < -k < -\log_2(3/2)$

$\Rightarrow \quad \log_2 3 > k > \log_2(3/2)$

$\Rightarrow \quad k < \log_2 3$

But $1 < \log_2 3 < 2$

$\Rightarrow \quad k < 2 \qquad \Rightarrow \quad k < 2$ and $k < 3$

(D) $\quad \sin\theta = \cos\varphi$

$\Rightarrow \quad \cos(90 - \theta) = \cos\varphi$

$\Rightarrow \quad 90° - \theta = 2n\pi \pm \varphi$

$\Rightarrow \quad \theta \pm \varphi - \dfrac{\pi}{2} = -2n\pi$

$\Rightarrow \quad \dfrac{\theta \pm \varphi - (\pi/2)}{\pi} = -2n = \text{even integer} = 0, 2$