PAPER - 1: PHYSICS, MATHEMATICS & CHEMISTRY

Test Booklet Code

H

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with *Blue/Black Ball point Pen. Use of pencil is strictly prohibited.*
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90 questions**. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of **Physics, Mathematics** and **Chemistry** having 30 questions in each part of equal weightage. Each questions is allotted **4 (four)** marks for correct response.
6. *Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. Use ***Blue / Black Ball Point Pen*** only for writing particulars / marking responses on **Side - 1** and **Side - 2** of the Answer sheet. ***Use of pencil is strictly prohibited.***
9. No candidate is allowed to carry any textual material, printed or written, bits of Papers, pager, mobile phone, any electronic device, etc. except and Admit Card inside the examination hall/ room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **3** pages (Pages **21-23**) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. ***However, the candidates are allowed to take away this Test Booklet with them***.
12. The CODE for this Booklet is **H**. Make sure that the CODE printed on **Side-2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray marks on the Answer Sheet.**

PART A – PHYSICS

1. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100ºC is : (For steel Young's modulus is 2×10^{11} Nm^{-2} and coefficient of thermal expansion is $1.1 \times 10^{-5} K^{-1}$)

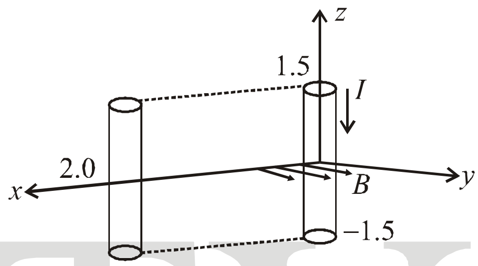
(1) 2.2×10^7 Pa (2) 2.2×10^6 Pa (3) 2.2×10^8 Pa (4) 2.2×10^9 Pa

Sol: [3] $\Delta L = L_0 \propto \Delta T$ and $y = \dfrac{PL_0}{\Delta L}$

$\Delta L = \Delta L$

$\Rightarrow \quad P = y \propto \Delta T = 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100 = 2.2 \times 10^{8}$

2. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4} e^{-0.2x} \hat{a}_y \; T$, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ m in 5×10^{-3} s. Assume parallel motion along the x-axis.



(1) 14.85 W (2) 29.7 W (3) 1.57 W (4) 2.97 W

Sol: [4] $P = \dfrac{W}{time} = \dfrac{\int F dx}{time} = \dfrac{\int_0^2 9 \times 10^{-3} e^{-0.2x} dx}{5 \times 10^{-3}} \approx 3$ W

3. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension

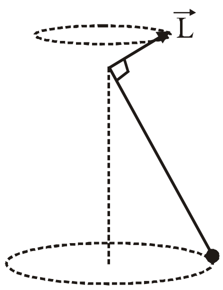
(1) angular momentum changes in direction but not in magnitude

(2) angular momentum changes both in direction and magnitude

(3) angular momentum is conserved

(4) angular momentum changes in magnitude but not in direction

Sol: [1] Angular momentum vector is constant in magnitude but changes in direction



4. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error mesuring ±0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?

(1) 0.5 mA (2) 0.05 mA (3) 0.2 mA (4) 0.02 mA

Sol: [3] $\frac{dI}{dV} = \frac{1000}{T} e^{\frac{1000V}{T}}$

$\Rightarrow \frac{dI}{dV} = \frac{1000}{T}\left(e^{\frac{1000V}{T}} - 1 + 1\right) \quad \Rightarrow \quad dI = \frac{1000}{300}(I+1)dV$

$\Rightarrow dI = \frac{1000}{300}(5+1) \times 0.01$ = 0.2 mA

5. An open glass tube is mmersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg)

(1) 38 cm (2) 6 cm (3) 16 cm (4) 22 cm

Sol: [3] $P_0 \times 8 = P \times x$ ...(*i*)

$P = P_0 - (54 - x)$ ...(*ii*)

$\Rightarrow$ $76 \times 8 = (22 + x)x$

$\Rightarrow$ $x = 16$ cm

6. Match **List-I** (Electromagnetic wave type) with **List-II** (Its association/application) and select the correct option from the choices given below the lists :

	List - I		**List - II**
(a)	Infrared waves	(i)	To treat muscular strain
(b)	Radio waves	(ii)	For broadcasting
(c)	X - rays	(iii)	To detect fracture of bones
(d)	Ultraviolet rays	(iv)	Absorbed by the ozone layer of the atmosphere

(1) a-iii, b-ii, c-i, d-iv (2) a-i, b-ii, c-iii, d-iv (3) a-iv, b-iii, c-ii, d-i (4) a-i, b-ii, c-iv, d-iii

Sol: [2]

7. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to

(1) 3×10^4 C/m^2 (2) 6×10^4 C/m^2 (3) 6×10^{-7} C/m^2 (4) 3×10^{-7} C/m^2

Sol: [3] $E = \frac{\sigma}{\epsilon_0 \epsilon_r} \quad \Rightarrow \quad \sigma = E\ \epsilon_0 \epsilon_r = \frac{3 \times 10^4 \times 2.2}{4\pi \times 9 \times 10^9} = 6 \times 10^{-7}$

8. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

(1) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm

(2) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

(3) A meter scale

(4) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm

Sol: [4] Least cannot is 0.01 cm.

L.C. of vernier = 1 msd – 1vsd = 0.01 cm

9. Four particles, each of mass *M* and equidistant from each other, move along a circle of radius *R* under the action of their mutual gravitational attraction. The speed of each particle is

(1) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$ (2) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$ (3) $\sqrt{\frac{GM}{R}}$ (4) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

Sol: [2] $\frac{mV^2}{R} = \frac{Gm^2}{\left(\sqrt{2}R\right)^2}\cos 45° \times 2 + \frac{Gm^2}{(2R)^2} \Rightarrow V = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

10. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be

(1) 12 A (2) 14 A (3) 8 A (4) 10 A

Sol: [1] P = 15 × 40 + 5 × 100 + 5 × 80 + 1 × 1000 = 3000W $\Rightarrow$ I = $\frac{2500}{220} = 13.6A$

11. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance *a* and in next τ s it travels 2*a*, in same direction, then

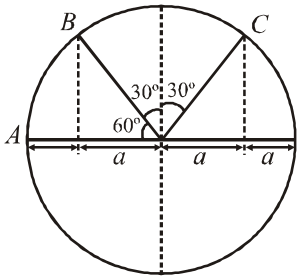
(1) amplitude of motion is 4*a*

(2) time period of oscillations is 6τ

(3) amplitude of motion is 3*a*

(4) time period of oscillations is 8τ

Sol: [2]



$T_{AB} = \tau$ and $T_{BC} = \tau$

Therefore amplitude = 2a and time period is 6τ

12. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid is

(1) 3 A (2) 6 A (3) 30 mA (4) 60 mA

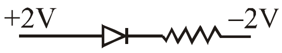
Sol: [1] $\because \quad H = \frac{B}{\mu_0} = (ni)$

$\Rightarrow \quad ni$ = coercitity $\quad \Rightarrow \quad \frac{100}{0.1} \times i = 3 \times 10^3 \quad \Rightarrow \quad i = 3$ A

13. The forward biased diode connection is

(1) 

(2) 

(3) 

(4) 

Sol: [3] For forward biasing *p*-type semiconductor must be at higher potential.

14. During the propagation of electromagnetic waves in a medium

(1) Electric energy density is equal to the magnetic energy density

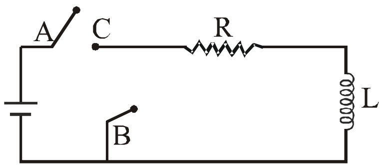
(2) Both electric and magnetic energy densities are zero

(3) Electric energy density is double of the magnetic energy density

(4) Electric energy density is half of the magnetic energy density

Sol: [1] Factual

15. In the circuit shown here, the point *C* is kept connected to point *A* till the current flowing through the circuit becomes constant. Afterward, suddenly, point *C* is disconnected from point *A* and connected to point *B* at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to



(1) –1 (2) $\frac{1-e}{e}$ (3) $\frac{e}{1-e}$ (4) 1

Sol: [1] Steady state current in inductor when *C* is connected to *A* is

$i_0 = \frac{V}{R}$ where *V* is emf of cell when *C* is connected to *B*. Current at

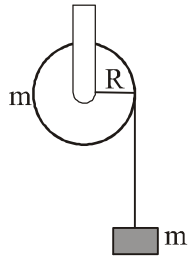
$i = \frac{V}{R} e^{-\frac{Rt}{L}}$

'OR'

Using Kirchhoff's loop rule at $t > 0$

$V_R + V_L = 0 \quad \Rightarrow \quad |V_R| / |V_L| = -1$

16. A mass m is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release?



(1) $\frac{5g}{6}$ (2) g

(3) $\frac{2g}{3}$ (4) $\frac{g}{2}$

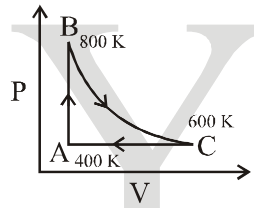
Sol: [4] Using $\tau = I\alpha$

$$mgR = (mR^2 + mR^2)\alpha$$

$$\Rightarrow \quad \alpha = \frac{g}{2R}$$

Acceleration of block = $R\alpha = \frac{g}{2}$

17. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement



(1) The change in internal energy in the process AB is –350 R

(2) The change in internal energy in the process BC is –500 R

(3) The change in internal energy in whole cyclic process is 250 R

(4) The change in internal energy in the process CA is 700 R

Sol: [2] Change in internal energy is given

$\Delta U = (n.C_v\Delta T)$

For process B to C

$$\Delta U = 1 \times \frac{5}{2}R(600 - 800)$$

$= \quad -500R$

18. From a tower of height H, a particle thrown vertically upwards with speed u. The time taken by the particle, hit the ground, is n times that taken by to reach the highest point of its path. The relation between H, u and n is

(1) $2gH = nu^2(n-2)$ (2) $gH = (n-2)u^2$ (3) $2gH = n^2u^2$ (4) $gH = (n-2)^2u^2$

Sol: [1] Let time is t_1 to reach ground

$$\Rightarrow \quad H = -ut_1 + \frac{1}{2}gt_1^2 \qquad ...(i)$$

Time to reach highest point

$$t_2 = \left(\frac{u}{g}\right)$$

$\because \quad t_1 = nt_2 = \dfrac{nu}{g}$

Puting t_1 in equation (i)

$$H = -\,u.\frac{nu}{g} + \frac{1}{2}g\left(\frac{nu}{g}\right)^2$$

$2gH = -2nu^2 + n^2u^2$

$2gH = nu^2(n - 2)$

19. A thin convex lens made from crown glass $\left(\mu = \dfrac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\dfrac{4}{3}$ and $\dfrac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is

(1) $f_2 > f$ and f_1 becomes negative
(2) f_1 and f_2 both become negative
(3) $f_1 = f_2 < f$
(4) $f_1 > f$ and f_2 becomes negative

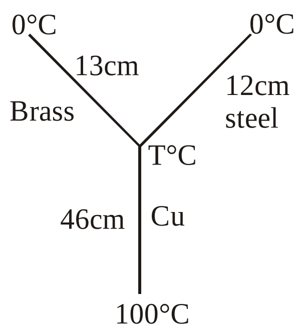
Sol: [4] Using $\dfrac{1}{f} = \left(\dfrac{\mu_2}{\mu_1} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

F_2 will be negative as $\mu_2\left(= \dfrac{3}{4}\right) < \mu_2\left(= \dfrac{5}{3}\right)$

20. Three rods of Copper, Brass and Steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100ºC where as ends of brass and steel are kept at 0ºC. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is

(1) 4.8 cal/s
(2) 6.0 cal/s
(3) 1.2 cal/s
(4) 2.4 cal/s

Sol: [1] Let temperature of function is T°C



$$\Rightarrow \quad \frac{0.92 \times A(100 - T)}{46} = \frac{0.26A(T - 0)}{13} + \frac{0.12A(T - 0)}{12}$$

$\Rightarrow \quad 2(100 - T) = 2(T) + T$

$\Rightarrow \quad 200 = 5T$

$T = 40°C$

21. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.

(1) 6
(2) 4
(3) 12
(4) 8

Sol: [1] Using $f = \frac{(2n+1)v}{4L}$

$$f_0 = \frac{v}{4L} = \frac{340}{4 \times 0.85} = 100 \text{ Hz}$$

Other possible frequencies are

f_1 = 300 Hz

f_2 = 500 Hz

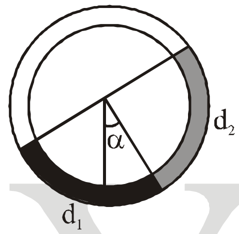
f_3 = 700 Hz

f_4 = 900 Hz

f_5 = 11 Hz

$\Rightarrow$ No of possible frequencies = 6

22. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90º angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is



(1) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (2) $\frac{1+\sin\alpha}{1-\cos\alpha}$ (3) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (4) $\frac{1+\cos\alpha}{1-\cos\alpha}$

Sol: [1] $(R\text{-}R \sin \alpha)d_1 g = (R\text{-}R \cos \alpha)d_1 g + (R \sin \alpha + R \cos \alpha)d_2 g$

$$\Rightarrow \frac{d_1}{d_2} = \frac{\sin\alpha + \cos\alpha}{(\cos\alpha - \sin\alpha)}$$

$$\frac{d_1}{d_2} = \frac{\tan\alpha + 1}{1 - \tan\alpha}$$

23. A green light is incident from the water to the air-water interface at the critical angle (θ). Select the **correct** statement.

(1) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium

(2) The entire spectrum of visible light will come out of the water at various angles to the normal

(3) The entire spectrum of visible light will come out of the water at an angle of 90º to the normal

(4) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium

Sol: [4] As wavelength increases (or frequency decreases) refractive index decreases and hence critical angle increases.

$\Rightarrow$ Less chance of internal reflection

24. Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionised Helium $(_2He^4)^+$ and double ionised lithium $(_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is **correct** ?

(1) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (2) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

(3) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (4) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

Sol: [1] $\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = Az^2$, where $A = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\Rightarrow \frac{1}{\lambda_1} = A(1)^2 \quad \text{or} \quad \lambda_1 = \frac{1}{A}$$

$$\frac{1}{\lambda_2} = A(1)^2 \quad \text{or} \quad \lambda_2 = \frac{1}{A}$$

$$\frac{1}{\lambda_3} = A(2)^2 \quad \text{or} \quad \lambda_3 = \frac{1}{4A}$$

$$\frac{1}{\lambda_4} = A(3)^2 \quad \text{or} \quad \lambda_4 = \frac{1}{9A}$$

Solving, $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

25. The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to

(1) 0.8 eV (2) 1.6 eV (3) 1.8 eV (4) 1.1 eV

Sol: [4] $KE_{max} = h\nu - \phi$

$$\frac{1}{1.6\times10^{-19}} \times \frac{r^2.q^2B^2}{2m} = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) - \phi$$

Solving, $\phi \approx 1.2eV$

26. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is

(1) $\frac{1}{3}$m (2) $\frac{1}{2}$m (3) $\frac{1}{6}$m (4) $\frac{2}{3}$m

Sol: [3] For No slipping

$mg \sin\theta = \mu\, mg \cos\theta$

$$\Rightarrow \mu = \tan\theta = \frac{dy}{dx} \quad \text{or} \quad \mu = \left(\frac{x^2}{2}\right)$$

$\Rightarrow \quad x = 1m$

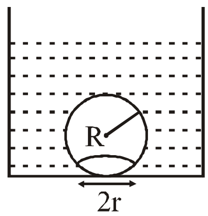
from, $y = \dfrac{x^2}{6}, y = \dfrac{1}{6}m$ = height of block from ground.

27. When a rubber-band is stretched by a distance *x*, it exerts a restoring force of magnitude $F = ax + bx^2$ where *a* and *b* are constants. The work done in stritching the unstretched rubber-band by *L* is

(1) $\dfrac{aL^2}{2} + \dfrac{bL^3}{3}$ (2) $\dfrac{1}{2}\left(\dfrac{aL^2}{2} + \dfrac{bL^3}{3}\right)$ (3) $aL^2 + bL^3$ (4) $\dfrac{1}{2}\left(aL^2 + bL^3\right)$

Sol: [1] $W = \int_0^L \left(ax + bx^2\right)dx = \left(\dfrac{ax^2}{2} + \dfrac{bx^3}{3}\right)_0^L = \dfrac{aL^2}{2} + \dfrac{bL^3}{3}$

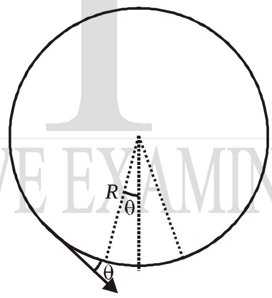
28. On heating water, bubbles being formed at the bottom of the vessel detatch and rise. Take the bubbles to be spheres of radius *R* and making a circular contact and radius *r* with the bottom of the vessel. If $r << R$, and the surface tension of water is *T*, value of *r* just before bubbles detatch is (density of water is ρ_w)



(1) $R^2\sqrt{\dfrac{\rho_w g}{T}}$ (2) $R^2\sqrt{\dfrac{3\rho_w g}{T}}$ (3) $R^2\sqrt{\dfrac{\rho_w g}{3T}}$ (4) $R^2\sqrt{\dfrac{\rho_w g}{6T}}$

Sol: [No Matching] To detach

upthrust = force due to surface tension



$\dfrac{4}{3}\pi R^3 \rho_w g = (T.\sin\theta).2\pi r$

$\Rightarrow \dfrac{4}{3}\pi R^3 \rho_w g = T.\dfrac{r}{R}.2\pi r$

$\Rightarrow \quad r = R^2\sqrt{\dfrac{2\rho_w g}{3T}}$

29. Two beams, *A* and *B*, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam *A* has maximum intensity (and beam *B* has zero intensity), a rotation of polaroid through 30º makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\dfrac{I_A}{I_B}$ equals

(1) 1 (2) $\dfrac{1}{3}$ (3) 3 (4) $\dfrac{3}{2}$

Sol: [2] $I_A.\cos^2 30 = I_B.\cos^2 60$

$\dfrac{I_A}{I_B} = \dfrac{1/4}{3/4} = \dfrac{1}{3}$

30. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is

(1) –80 J (2) 80 J (3) 120 J (4) –120 J

Sol: [1] $\int_{V_O}^{V_A} dV = -\int_O^2 30x^2 dx$

$V_A - V_0 = -30\left(\frac{x^3}{3}\right)_0^2$ = – 10[8 – 0] = – 80 volt.

PART B – MATHEMATICS

31. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line:

(1) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$ (2) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

(3) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$ (4) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

Sol: [1] The image of the point (1, 3, 4) on the plane is (–3, 5, 2)

so the equation of image is $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

32. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

(1) $\left(16, \frac{251}{3}\right)$ (2) $\left(14, \frac{251}{3}\right)$ (3) $\left(14, \frac{272}{3}\right)$ (4) $\left(16, \frac{272}{3}\right)$

Sol: [4] Cofficient of x^3 = ${}^{18}C_3 .(-2)^3 + a. {}^{18}C_2. (-2)^2 + b\ {}^{18}C_1. (-2) = 0$

$\Rightarrow 51a - 3b = 17 \times 32$ ....(*i*)

Cofficient of x^4 = ${}^{18}C_4 (-2)^4 + a\ {}^{18}C_3(-2)^3 + b\ {}^{18}C_2(-2)^2 = 0$

$\Rightarrow -32a + 3b = -16 \times 15$ ....(*ii*)

Solving (*i*) and (*ii*)

$a = 16, b = \frac{272}{3}$

33. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval:

(1) $(-1, 0) \cup (0, 1)$ (2) $(1, 2)$ (3) $(-2, -1)$ (4) $(-\infty, -2) \cup (2, \infty)$

Sol: [1] $-3\{x\}^2 + 2\{x\} + a^2 = 0$

$\Rightarrow a^2 = 3\{x\}^2 - 2\{x\}$

let $\{x\} = t$

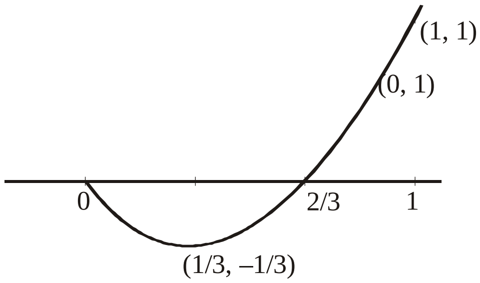
As x is not an integer $\therefore\ 0 < t < 1$

$\therefore\ a^2 = 3t^2 - 2t$

$a^2 = 3t\left(t - \frac{2}{3}\right)$

let $f(t) = 3t\left(t - \frac{2}{3}\right)$

$f(t)_{min} = = -\frac{1}{3}$ at $t = \frac{1}{3}$



As $0 < t < 1$

$\therefore \quad -\frac{1}{3} < f(t) < 1$

$\Rightarrow -\frac{1}{3} < a^2 < 1$

$\Rightarrow a \in (-1, 0) \cup (0, 1)$

34. If $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = \lambda\ [\vec{a}\vec{b}\vec{c}]^2$ then λ is equal to

(1) 2 (2) 3 (3) 0 (4) 1

Sol: [4] $\because |\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}| = |\vec{a}\,\vec{b}\vec{c}|^2$

so $\lambda = 1$

35. The variance of first 50 even natural numbers is

(1) $\frac{833}{4}$ (2) 833 (3) 437 (4) $\frac{437}{4}$

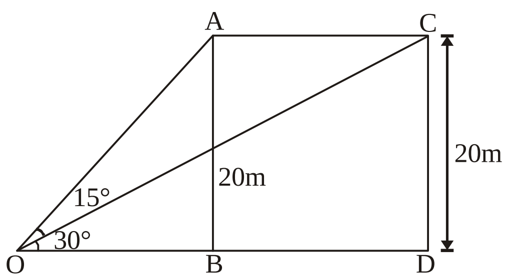
Sol: [2] Var. $= \frac{(2-51)^2 + (4-51)^2 + ... + (100-51)^2}{50} = 833$

36. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30°. Then the speed (in m/s) of the bird is

(1) $40(\sqrt{2}-1)$ (2) $40(\sqrt{3}-\sqrt{2})$ (3) $20\sqrt{2}$ (4) $20(\sqrt{3}-1)$

Sol: [4] OB = AB = 20 m

$\Rightarrow$ AC = $(20\sqrt{3} - 20) = 20(\sqrt{3} - 1)$ m/s



37. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ is equals

(1) $\pi - 4$ (2) $\frac{2\pi}{3}-4-4\sqrt{3}$ (3) $4\sqrt{3}-4$ (4) $4\sqrt{3}-4-\frac{\pi}{3}$

Sol: [4] $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx = \int_0^{\pi}\left|1-2\sin\frac{x}{2}\right|dx$

$= 2\int_0^{\pi/2}|1-2\sin t|\,dt = 2\int_0^{\pi/6}|1-2\sin t|\,dt + 2\int_0^{\pi/2}(2\sin t-1)\,dt$

$= 4\sqrt{3}-4-\frac{\pi}{3}$

38. The statement $\sim(p \leftrightarrow \sim q)$ is

(1) equivalent to $p\leftrightarrow q$ (2) equivalent to $\sim p\leftrightarrow q$ (3) a tautology (4) a fallacy

Sol: [1] Making truth table

$\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

39. If A is an 3 × 3 non-singular matrix such that $AA' = A'A$ and $B' = A^{-1}A'$, then BB' equals

(1) 1 + B (2) 1 (3) B^{-1} (4) $(B^{-1})'$

Sol: [2] $B' = (A^{-1}A')' = A(A^{-1})'$

$BB' = A^{-1}A'A(A^{-1})'$

$= A^{-1}AA'(A^{-1})' = I$

40. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

(1) $(x-1)e^{x+\frac{1}{x}}+c$ (2) $xe^{x+\frac{1}{x}}+c$ (3) $(x+1)e^{x+\frac{1}{x}}+c$ (4) $-xe^{x+\frac{1}{x}}+c$

Sol: [2] $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$

$= \int e^{x+\frac{1}{x}}dx + \int\left(x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$

$= xe^{x+\frac{1}{x}} - \int\left(x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx + \int\left(x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx + c$

$= xe^{x+\frac{1}{x}} + c$

41. If z is a complex number such that $|x| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$

(1) is equal to $\frac{5}{2}$

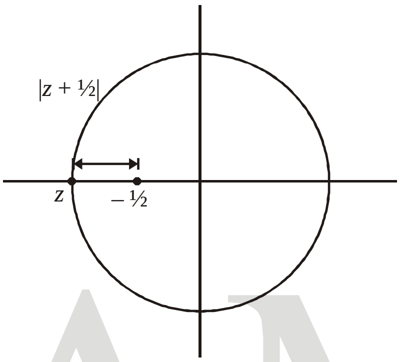
(2) lies in the interval (1, 2)

(3) is strictly greater than $\frac{5}{2}$

(4) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

Sol: [2] From diagram

$$\left|z+\frac{1}{2}\right|_{\min} = \frac{3}{2}$$



42. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to

(1) $1+x^5$

(2) $5x^4$

(3) $\frac{1}{1+\{g(x)\}^5}$

(4) $1+\{g(x)\}^5$

Sol: [4] $fog = x$

$f'(g(x)) \times g'(x) = 1 \Rightarrow g'(x) = \frac{1}{f'(g(x))}$

$\Rightarrow g'(x) = 1 + g(x)^5$

43. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then

K is equal to

(1) $\alpha\beta$

(2) $\frac{1}{\alpha\beta}$

(3) 1

(4) –1

Sol: [3] $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$

$= (\alpha-1)^2 \ (\beta-1)^2 = (\alpha-\beta)^2$ Hence $k = 1$.

44. Let $f_K(x) = \frac{1}{K}$ $(\sin^k x + \cos^k x)$ where $x \in$ R and K $\geq$ 1. Then $f_4(x) - f_6(x)$ equals:

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{12}$

Sol: [4] $f_4(x) - f_6(x) = \frac{1}{4}[\sin^4 x + \cos^4 x] - \frac{1}{6}[\sin^6 x + \cos^6 x]$

$$= \frac{1}{12}[\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x] = \frac{1}{12}$$

45. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4,$ then the value of $|\alpha - \beta|$ is

(1) $\frac{\sqrt{61}}{9}$ (2) $\frac{2\sqrt{17}}{9}$ (3) $\frac{\sqrt{34}}{9}$ (4) $\frac{2\sqrt{13}}{9}$

Sol: [4] $\frac{\alpha + \beta}{\alpha\beta} = 4$

$\Rightarrow \alpha + \beta = \frac{-q}{p}, \alpha\beta = \frac{\gamma}{p}$

$\Rightarrow q = -4r$ and $2q = p + r \Rightarrow p = -9r$

$\Rightarrow px^2 + qx + r = 9x^2 + 4x - 1 = 0$

$\Rightarrow |\alpha - \beta| = \frac{\sqrt{52}}{9} = \frac{2\sqrt{13}}{9}$

46. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are:

(1) mutually exclusive and independent.
(2) equally likely but not independent.
(3) independent but not equally likely
(4) independent and equally likely.

Sol: [3] $P(A \cup B) = \frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$

$\Rightarrow P(B) = \frac{1}{3}$

$P(A) \times P(B) = P(A \cap B) = \frac{1}{4}$

and $P(A) = \frac{3}{4} \neq P(B)$

47. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$:

(1) $2f'(c) = g'(c)$ (2) $2f'(c) = 3g'(c)$ (3) $f'(c) = g'(c)$ (4) $f'(c) = 2g'(c)$

Sol: [4] For some $c \in]0, 1[$

$$f'(c) = \frac{f(1) - f(0)}{1-0} = \frac{6-2}{1} = 4$$

and $g'(c) = \dfrac{g(1) - g(0)}{1-0} = \dfrac{2-0}{1} = 2$

$\Rightarrow 2g'(c) = f'(c)$

48. Let the population of rabbits surviving at a time t be governed by the differential equation $\dfrac{dp(t)}{dt} = \dfrac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ equals:

(1) $400 - 300\, e^{t/2}$ (2) $300 - 200\, e^{-t/2}$ (3) $600 - 500\, e^{t/2}$ (4) $400 - 300\, e^{-t/2}$

Sol: [1] $\dfrac{dp}{dt} - \dfrac{p}{2} = -200$

Integrating factor = $e^{\int -\frac{1}{2}dt} = e^{-\frac{t}{2}}$

$\Rightarrow pe^{-\frac{t}{2}} = \int 200e^{-\frac{t}{2}}dt + c$

$\Rightarrow pe^{-\frac{t}{2}} = 400e^{-\frac{t}{2}} + c$

put $t = 0$ and $p = 100 \Rightarrow c = -300$

$\Rightarrow p = 400 - 300\, e^{t/2}$

49. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centrad at (0, y) passing through origin and touching the circle C externally, then the radius of T is equal to

(1) $\dfrac{\sqrt{3}}{\sqrt{2}}$ (2) $\dfrac{\sqrt{3}}{2}$ (3) $\dfrac{1}{2}$ (4) $\dfrac{1}{4}$

Sol: [4] $C_1C_2 = r_1 + r_2$

$\sqrt{1 + (y-1)^2} = 1 + y$

$\Rightarrow y = \dfrac{1}{4}$ = radius

50. The area of the region described by A = {$(x,y) : x^2 + y^2 \le 1$ and $y^2 \le 1 - x$} is:

(1) $\dfrac{\pi}{2} + \dfrac{4}{3}$ (2) $\dfrac{\pi}{2} - \dfrac{4}{3}$ (3) $\dfrac{\pi}{2} - \dfrac{2}{3}$ (4) $\dfrac{\pi}{2} + \dfrac{2}{3}$

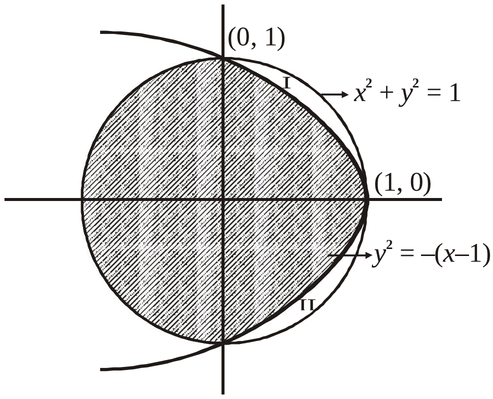
Sol: [1] Area I = area II (by symmetry)

$\therefore$ The required area = π –2area I

$= \pi - 2\left[\int_0^1 (\sqrt{1-x^2} - \sqrt{1-x})dx\right]$

$= \pi - 2\left[\left|\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right|_0^1 - \int_0^1 \sqrt{1-x}dx\right]$

$= \pi - 2\left[\frac{\pi}{4} - \frac{2}{3}\right] = \frac{\pi}{2} + \frac{4}{3}$



51. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then

(1) $2bc - 3ad = 0$ (2) $2bc + 3ad = 0$ (3) $3bc - 2ad = 0$ (4) $3bc + 2ad = 0$

Sol: [3] Point of intersection is $(h, -h)$ which lies on $x + y = 0$

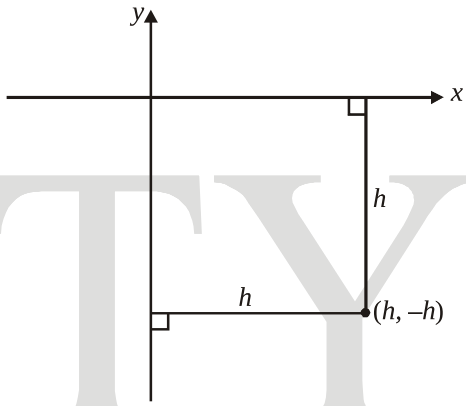
The two lines are $5x + 2y = -\frac{d}{b}$

$\Rightarrow 4x + 2y = -\frac{c}{a}$

Putting $x + y = 0$ in 2 equations.

we get $3x = -\frac{d}{b}$ and $2x = -\frac{c}{a}$

$\Rightarrow \frac{-d}{3b} = \frac{-c}{2a} \Rightarrow 3bc = 2ad$



52. Let PS be the median of the triangle with vertices P(2, 2), Q(6, –1) and R(7, 3). The equation of the line passing through (1, –1) and parallel to PS is

(1) $4x - 7y - 11 = 0$ (2) $2x + 9y + 7 = 0$ (3) $4x + 7y + 3 = 0$ (4) $2x - 9y - 11 = 0$

Sol: [2] S $\equiv$ mid point of QR $\equiv$ (6.5, 1)

Slope of PS = $\frac{1-2}{6.5-2} = \frac{-1}{4.5}$

The equation of line is

$y + 1 = -\frac{2}{9}(x-1)$

$\Rightarrow 9y + 2x + 7 = 0$

53. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to

(1) $\frac{\pi}{2}$ (2) 1 (3) $-\pi$ (4) π

Sol: [4] $\lim_{x\to 0}\frac{\sin[\pi(1-\sin^2 x)]}{x^2}$

$= \lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{(\pi\sin^2 x)}\frac{\pi\sin^2 x}{x^2}$

$= 1 \times \pi = \pi$

54. If X = $\{4^n - 3n - 1 : n \in \mathrm{N}\}$ and Y = $\{9(n-1) : n \in \mathrm{N}\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to

(1) N (2) Y – X (3) X (4) Y

Sol: [4] $X = 4^n - 3n - 1$
$= (1 + 3)^n - 3n - 1$
$= 1 + 3n + 3^2({}^nC_2 + {}^nC_3 .\ 3 + ...) - 3n - 1 = 9\ K$
for $n = 1$, starting value of X = 0
So X contains multiples of 9 starting from O
Y contains all multiples of 9 starting from O
$\Rightarrow X \cup Y = Y$

55. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is

(1) $(x^2 - y^2)^2 = 6x^2 + 2y^2$ (2) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

(3) $(x^2 + y^2)^2 = 6x^2 + 2y^2$ (4) $(x^2 + y^2)^2 = 6x^2 - 2y^2$

Sol: [3] The ellipse is $\frac{x^2}{6}+\frac{y^2}{2}=1$

Any tangent is $\frac{x\cos\theta}{\sqrt{6}}+\frac{y\sin\theta}{\sqrt{2}}=1$ ...(*i*)

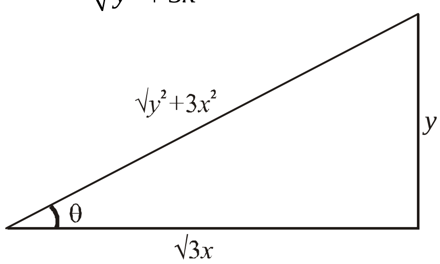
⊥r to it passing through origin is $\frac{x\sin\theta}{\sqrt{2}}-\frac{y\cos\theta}{\sqrt{6}}=0$ ...(*ii*)

Solving them, $x=\frac{2\sqrt{6}\cos\theta}{2\cos^2\theta+6\sin^2\theta}$, $y=\frac{6\sqrt{2}\sin\theta}{2\cos^2\theta+6\sin^2\theta}$

Dividing, $\frac{y}{x}=\sqrt{3}\tan\theta \Rightarrow \cos\theta = \frac{\sqrt{3}x}{\sqrt{y^2+3x^2}}, \sin\theta=\frac{y}{\sqrt{y^2+3x^2}}$

Squaring and adding (*i*) and (*ii*),

$(x^2 + y^2)\left(\frac{\cos^2\theta}{6}+\frac{\sin^2\theta}{2}\right)=1$



$\Rightarrow \frac{(x^2+y^2)}{(y^2+3x^2)}\left(\frac{x^2}{2}+\frac{y^2}{2}\right)=1$

$\Rightarrow (x^2 + y^2)^2 = 6x^2 + 2y^2$

56. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is

(1) $\sqrt{2}+\sqrt{3}$ (2) $3+\sqrt{2}$ (3) $2-\sqrt{3}$ (4) $2+\sqrt{3}$

Sol: [4] Let G.P. be a, ar, ar^2.
Then $a, 2ar, ar^2$ are in A.P.
$\Rightarrow a + ar^2 = 4ar$
$r^2 - 4r + 1 = 0$
$\Rightarrow r = \dfrac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$
Since G.P. is increasing, $r > 1$
$\Rightarrow r = 2 + \sqrt{3}$

57. If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + \ldots + 10(11)^9 = k (10)^9$, then k is equal to

(1) $\dfrac{121}{10}$ (2) $\dfrac{441}{100}$ (3) 100 (4) 110

Sol: [3] $S = 10^9 + 2(11)^1(10)^8 + 3(11)^2 (10)^8 + \ldots + 10(11)^9$

$\dfrac{11}{10}S = (11)^1 10^8 + 2\, 11^2 10^7 + \ldots + (11)^{10}$

Subtracting

$$-\frac{1}{10}S = \underbrace{(10^9 + 11\times 10^8 + 11^2\times 10^7 + \ldots + 11^9)}_{\text{G.P.}} - 11^{10}$$

$$= \frac{10^9\left(1-\left(\frac{11}{10}\right)^{10}\right)}{1-\frac{11}{10}} - 11^{10} = -10^{10}$$

$\Rightarrow S = 10^{11} = 100.\ (10)^9$

58. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is

(1) $\dfrac{\pi}{3}$ (2) $\dfrac{\pi}{4}$ (3) $\dfrac{\pi}{6}$ (4) $\dfrac{\pi}{2}$

Sol: [1] $l^2 + m^2 + n^2 = 1$

and $m^2 + n^2 = l^2$

gives $2l^2 = 1 \Rightarrow l^2 - \dfrac{1}{2} = 0 \Rightarrow l_1 l_2 = -\dfrac{1}{2}$

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$
Now $m + n = -l$
$\Rightarrow (m + n)^2 = l^2 \Rightarrow m^2 + n^2 + 2mn = l^2$.
$\Rightarrow mn = 0$
For I[st] line, let $m = 0$ *i.e.* $m_1 = 0$

for IInd line, let $n = 0$ *i.e.*, $n_2 = 0$

$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$

$= -\frac{1}{2} + 0 + 0$

$= -\frac{1}{2}$

$\Rightarrow \theta = \frac{2\pi}{3}$

another angle $\alpha = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

59. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{1}{8}$ (4) $\frac{2}{3}$

Sol: [1] Any tangent to $y^2 = 4x$ is

$y = mx + \frac{1}{m}$

$x^2 = -32\left(mx + \frac{1}{m}\right)$

$\Rightarrow x^2 + 32\,mx + \frac{32}{m} = 0$

$D = 0 \Rightarrow m = \frac{1}{2}$

60. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$ then

(1) $\alpha = -6, \beta = \frac{1}{2}$ (2) $\alpha = -6, \beta = -\frac{1}{2}$ (3) $\alpha = 2, \beta = -\frac{1}{2}$ (4) $\alpha = 2, \beta = \frac{1}{2}$

Sol: [3] $f'(x) = \frac{\alpha}{x} + 2\beta x + 1 = \frac{\alpha + 2\beta x^2 + x}{x} = 0$

$\Rightarrow x = -1, 2$

$S = \frac{-1}{2\beta} = 1 \Rightarrow \beta = \frac{-1}{2}$

$P = \frac{\alpha}{2\beta} = -2 \Rightarrow \alpha = 2$

PART C – CHEMISTRY

61. Which one of the following properties is not shown by NO?

(1) It combines with oxygen to form nitrogen dioxide

(2) It's bond order is 2.5

(3) It is diamagnetic in gaseous state

(4) It is a neutral oxide

Sol: [3] NO in gaseous state exist as monomeric form and is an odd electron molecule. Hence it is paramagnetic.

62. If Z is a compressibility factor, van der Waals equation at low pressure can be written as:

(1) $Z = 1 - \frac{Pb}{RT}$ (2) $Z = 1 + \frac{Pb}{RT}$ (3) $Z = 1 + \frac{RT}{Pb}$ (4) $Z = 1 - \frac{a}{VRT}$

Sol: [4] $\left(\rho + \frac{a}{V^2}\right)(V - b) = RT$

as low P; $(V - b) \cong V$

$\left(P + \frac{a}{V^2}\right)V = RT$; $PV = \frac{a}{V} = RT$

$\frac{PV}{RT} = \frac{RT}{RT} - \frac{a}{VRT}$

$Z = 1 - \frac{a}{VRT}$

63. The metal that cannot be obtained by electrolysis of an aqueous solution of its salts is

(1) Cu (2) Cr (3) Ag (4) Ca

Sol: [4]

64. Resistance of 0.2 M solution of an electrolyte is 50Ω. The specific conductance of the solution is 1.4 S m^{-1}. The resistance of 0.5 M solution of the same electrolyte is 280 Ω. The molar conductivity of 0.5M solution of the electrolyte in S m^2 mol^{-1} is

(1) 5×10^3 (2) 5×10^2 (3) 5×10^{-4} (4) 5×10^{-3}

Sol: [3] $K = \rho\frac{\ell}{a}$

$1.4 = \frac{1}{50} \times \frac{\ell}{a}$

$\frac{\ell}{a} = 50 \times 1.4 m^{-1}$

$K = \rho\frac{\ell}{a} = \frac{1}{280} \times 50 \times 1.4 = \frac{1}{4}$

$\lambda_m = \frac{K \times 10^{-3}}{M} = \frac{1 \times 10^{-3}}{4 \times 0.5}$ = 5×10^{-4} Sm^2 mol^{-1}

65. CsCl crystallises in body centred cubic lattice. If 'a' is its edge length then which of the following expressions is correct?

(1) $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}a$ (2) $r_{Cs^+} + r_{Cl^-} = \sqrt{3}a$ (3) $r_{Cs^+} + r_{Cl^-} = 3a$ (4) $r_{Cs^+} + r_{Cl^-} = \frac{3a}{2}$

Sol: [1] $2r_{Cs^+} + 2r_{Cl^-} = \sqrt{3}a$ $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}$

66. Consider separate solutions of 0.500 M C_2H_5OH(aq), 0.100 M $Mg_3(PO_4)_2$(aq), 0.250 M KBr(aq) and 0.125 M Na_3PO_4(aq) at 25°C. Which statement is true about these solutions, assuming all salts to be strong electrolytes?

(1) 0.125 M Na_3PO_4(aq) has the highest osmotic pressure

(2) 0.500 M C_2H_5OH(aq) has the highest osmotic pressure

(3) They all have the same osmotic pressure

(4) 0.100 M $Mg_3(PO_4)_2$(aq) has the highest osmotic pressure

Sol: [3] π (osmotic pressure) = i ×CRT

67. In which of the following reactions H_2O_2 acts as a reducing agent?

(a) $H_2O_2 + 2H^+ + 2e^- \rightarrow 2H_2O$ (b) $H_2O_2 - 2e^- \rightarrow O_2 + 2H^+$

(c) $H_2O_2 + 2e^- \rightarrow 2OH^-$ (d) $H_2O_2 + 2OH^- - 2e^- \rightarrow O_2 + 2H_2O$

(1) (a), (c) (2) (b), (d) (3) (a), (b) (4) (c), (d)

Sol: [2] b & d

in both reaction oxidation no. of oxygen changes from – 1 to 0.

68. In S_N2 reactions, the correct order of reactivity for the following compounds:

CH_3Cl, CH_3CH_2Cl, $(CH_3)_2CHCl$ and $(CH_3)_3CCl$ is

(1) $CH_3CH_2Cl > CH_3Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

(2) $(CH_3)_2CHCl > CH_3CH_2Cl > CH_3Cl > (CH_3)_3CCl$

(3) $CH_3Cl > (CH_3)_2CHCl > CH_3CH_2Cl > (CH_3)_3CCl$

(4) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

Sol: [4]

69. The octahedral complex of a metal ion M^{3+} with four monodentate ligands L_1, L_2, L_3 and L_4 absorb wavelengths in the region of red, green, yellow and blue, respectively. The increasing order of ligand strength of the four ligands is

(1) $L_3 < L_2 < L_4 < L_1$ (2) $L_1 < L_2 < L_4 < L_3$ (3) $L_4 < L_3 < L_2 < L_1$ (4) $L_1 < L_3 < L_2 < L_4$

Sol: [4] The wavelenght red, green, yellow and blue is increases in the order of red, yellow, green and blue.

70. For the estimation of nitrogen, 1.4 g of an organic compound was digested by Kjeldahl method and the evolved ammonia was absorbed in 60 mL of M/10 sulphuric acid. The unreacted acid required 20 mL of M/10 sodium hydroxide for complete neutralization. The percentage of nitrogen in the compound is

(1) 3% (2) 5% (3) 6% (4) 10%

Sol: [4] % of N $= \dfrac{1.4 \times M \times 2(V - V_1/2)}{M}$

M = molarity of H_2SO_4

V = volume of H_2SO_4

V_1 = volume of NaOH

m = mass of organic compound

% of $N = \dfrac{1.4 \times 0.1 \times 2(60 - 20/2)}{1.4} = 100 \times 0.1 = 10\%$

71. The equivalent conductance of NaCl at concentration C and at infinite dilution are λ_C and λ_∞, respectively. The correct relationship between λ_C and λ_∞ is given as (where the constant B is positive)

(1) $\lambda_C = \lambda_\infty - (B)\sqrt{C}$ (2) $\lambda_C = \lambda_\infty + (B)\sqrt{C}$ (3) $\lambda_C = \lambda_\infty + (B)C$ (4) $\lambda_C = \lambda_\infty - (B)C$

Sol: [1] $\lambda_C = \lambda_\infty - B\sqrt{C}$

72. For the reaction $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$ if $K_P = K_C(RT)^x$ where the symbols have usual meaning then the value of x is (assuming ideality)

(1) $\frac{1}{2}$ (2) 1 (3) -1 (4) $-\frac{1}{2}$

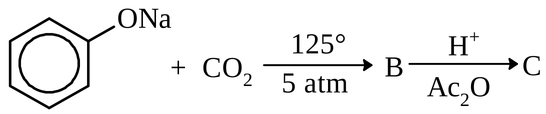
Sol: [4] $Kp = K_C(RT)^{\Delta n_g}$ $\Delta n = 1 - (1 + ½)$ $= -1/2$.

73. In the reaction $CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCl_5} B \xrightarrow{Alc.\ KOH} C$, the product C is

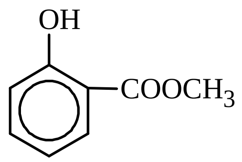
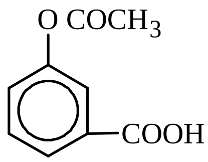
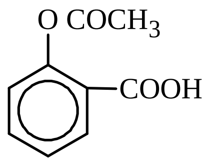
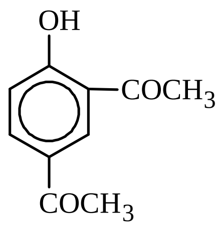
(1) Ethylene (2) Acetyl chloride (3) Acetaldehyde (4) Acetylane

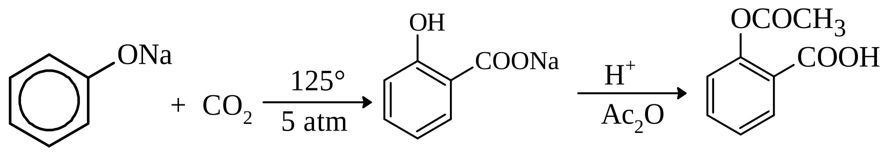
Sol: [1] $CH_3 - COOH \xrightarrow{LiAlH_4} CH_3CH_2OH \xrightarrow{PCl_5} CH_3CH_2Cl \xrightarrow{Alc.KOH} CH_2 = CH_2$

74. Sodium phenoxide when heated with CO_2 under pressure at 125°C yields a product which on acetylation produces C.



The major product C would be

(1)  (2)  (3)  (4) 

Sol: [3] 

75. On heating an aliphatic primary amine with chloroform and ethanolic potassium hydroxide, the organic compound formed is

(1) an alkyl cyanide (2) an alkyl isocyanide (3) an alkanol (4) an alkanediol

Sol: [2] $R-NH_2 + CHCl_3 + 3KOH \longrightarrow \underset{\text{Alkyl isocyanide}}{R-\overset{+}{N}\equiv\overset{-}{C}} + 3KCl + 3H_2O$

76. The correct statement for the molecule, CsI_3, is

(1) it contains Cs^{3+} is I^- ions (2) is contains Cs^+, I^- and lattice I_2 molecule

(3) it is a covalent molecule (4) it contains Cs^+ and I_3^- ions

Sol: [4] CsI_3contain Cs^+ and I_3^- ions.

77. The equation which is balanced and represents the correct product(s) is

(1) $[Mg(H_2O)_6]^{2+} + (EDTA)^{4-} \xrightarrow{\text{excess NaOH}} [Mg(EDTA)]^{2+} + 6H_2O$

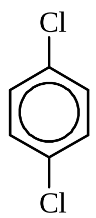
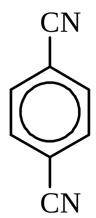
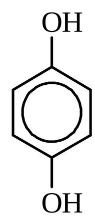
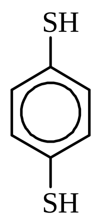
(2) $CuSO_4 + 4KCN \longrightarrow K_2[Cu(CN)_4] + K_2SO_4$

(3) $Li_2O + 2KCl \longrightarrow 2LiCl + K_2O$

(4) $[CoCl(NH_3)_5]^+ + 5H^+ \longrightarrow Co^{2+} + 5NH_4^+ + Cl^-$

Sol: [4]

78. For which of the following molecule significant $\mu \neq 0$?

(a)  (b)  (c)  (d) 

(1) only (c) (2) (c) and (d) (3) only (a) (4) (a) and (b)

Sol: [2]

79. For the non-stoichiometre reaction $2A + B \rightarrow C + D$, the following kinetic data were obtained in three separated experiments, all at 298 K

Initial Concentration (A)	Initial Concentration (B)	Initial rate of formation of C (mol L^{-1} S^{-})
0.1 M	0.1 M	1.2×10^{-3}
0.1 M	0.2 M	1.2×10^{-3}
0.2 M	0.1 M	2.4×10^{-3}

The rate law for the formation of C is

(1) $\frac{dc}{dt} = k[A][B]^2$ (2) $\frac{dc}{dt} = k[A]$ (3) $\frac{dc}{dt} = k[A][B]$ (4) $\frac{dc}{dt} = k[A]^2[B]$

Sol: [2] $r_1 = 1.2 \times 10^{-3} = K[0.1]^x[0.1]^y$

$r_2 = 1.2 \times 10^{-3} = K[0.1]^x[0.2]^y$

$r_3 = 2.4 \times 10^{-3} = K[0.2]^x[0.1]^y$

$\frac{r_3}{r_2} = \frac{2.4 \times 10^{-3}}{1.2 \times 10^{-3}} = \frac{K[0.2]^x[0.1]^y}{K[0.1]^x[0.2]^y}$ $2^1 = 2^x.2^{-y}$ $x - y = 1$

$\frac{r_2}{r_1} = \frac{1.2 \times 10^{-3}}{1.2 \times 10^{-3}} = \frac{K[0.1]^x[0.2]^y}{K[0.1]^x[0.1]^y} = 2^y$ $2^y = 1,\ y = 0$

$x = 1$ $r = K[A]$

80. Which series of reactions correctly represent chemical relations related to iron and its compound?

(1) $Fe \xrightarrow{Cl_2,\ heat} FeCl_3 \xrightarrow{heat,\ air} FeCl_2 \xrightarrow{Zn} Fe$

(2) $Fe \xrightarrow{O_2,\ heat} Fe_3O_4 \xrightarrow{CO, 600°C} FeO \xrightarrow{CO, 700°C} Fe$

(3) $Fe \xrightarrow{dill\ H_2SO_4} FeSO_4 \xrightarrow{H_2SO_4,\ O_2} Fe_2(SO_4)_3 \xrightarrow{heat} Fe$

(4) $Fe \xrightarrow{O_2,\ heat} FeO \xrightarrow{dill\ H_2SO_4} FeSO_4 \xrightarrow{heat} Fe$

Sol: [2] $Fe \xrightarrow{O_2,\ heat} Fe_3O_4 \xrightarrow{CO, 600°C} FeO \xrightarrow{Co, 700°C} Fe$

81. Considering the basic strength of amines in aqueous solution, which one has the smallest pKb value?

(1) $(CH_3)_3N$ (2) $C_6H_5NH_2$ (3) $(CH_3)_2NH$ (4) CH_3NH_2

Sol: [3] Basic strength of following amines in aqueous solution is
$(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N > C_6H_5NH_2$

82. Which one of the following bases is not present in DNA?

(1) Cytosine (2) Thymine (3) Quinoline (4) Adenine

Sol: [3] Nitrogenous bases present in DNA are

(i) Purines	**(ii) Pyrimidines**
(a) Adenine	(a) Thymine
(b) Guanine	(b) Cytosine

83. The correct set of four quantum numbers for the valence electrons of rubidium atom (Z = 37) is

(1) $5, 1, 1, +\frac{1}{2}$ (2) $5, 0, 1, +\frac{1}{2}$ (3) $5, 0, 0, +\frac{1}{2}$ (4) $5, 1, 0, +\frac{1}{2}$

Sol: [3] Rb = [Kr] $5s^1$ n = 5, l = 0, m = 0, s = +½

84. The major organic compound formed by the reaction of 1, 1, 1, trichloroethane with silver powder is

(1) 2- Butyne (2) 2-Butene (3) Acetylene (4) Ethene

Sol: [1] $$CH_3-C\begin{bmatrix}Cl\\Cl\\Cl\end{bmatrix} + 6\ Ag + \begin{bmatrix}Cl\\Cl\\Cl\end{bmatrix}C-CH_3 \xrightarrow[+\ \Delta]{-6\ AgCl} H_3C-C\equiv C-CH_3$$

85. Given below are the half-cell reaction:

$Mn^{2+} + 2e^- \rightarrow Mn$, E° = – 1.18 V

$2(Mn^{3+} + e^- \rightarrow Mn^{2+})$; E° = + 1.51 V

The E° for $3Mn^{2+} \rightarrow Mn + 2Mn^{3+}$ will be

(1) –0.33 V; the reaction will not occur (2) – 0.33 V; the reaction will occur

(3) – 2.69 V; the reaction will not occur (4) – 2.69 V; the reaction will occur

Sol: [3] $E^o_{cell} = E^o_{cathode} - E^o_{anode} = -1.18 - (+1.51) = -2.69\,V$

Since E^o_{cell} is –ve, hence reaction is not possible.

86. The ratio of masses of oxygen and nitrogen in a particular gaseous mixture is 1 : 4. The ratio of number of their molecule is

(1) 1 : 8 (2) 3 : 16 (3) 1 : 4 (4) 7 : 32

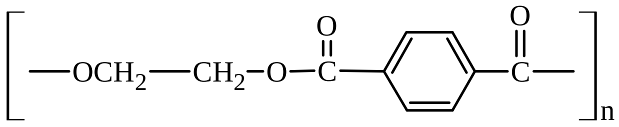
Sol: [4] Ratio of no. of molecules = ratio of no. of moles

$$\frac{n_{O_2}}{n_{N_2}} = \frac{W_{O_2}}{M_{O_2}} \times \frac{M_{N_2}}{W_{N_2}} = \frac{1}{32} \times \frac{28}{4} = 7 : 32$$

87. Which one is classified as a condensation polymer?

(1) Teflon (2) Acrylonitrile (3) Dacron (4) Neoprene

Sol: [3] Dacron is condensation polymer of ethylene glycol and terepthalic acid.



88. Among the following oxoacids, the correct decreasing order of acid strength is

(1) $HClO_4 > HClO_3 > HClO_2 > HOCl$ (2) $HClO_2 > HClO_4 > HClO_3 > HOCl$

(3) $HOCl > HClO_2 > HClO_3 > HClO_4$ (4) $HClO_4 > HOCl > HClO_2 > HClO_3$

Sol: [1] Acidic strength of oxoacids of chlorine is in the order of $HClO_4 > HClO_3 > HClO_2 > HClO$

89. For complete combustion of ethanol,

$$C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l),$$

the amount of heat produced as measured in bomb calorimeter, is 1364.47 kJ mol^{-1} at 25°C. Assuming ideality the enthalpy of combustion, $\Delta_C H$, for the reaction will be (R = 8.314 kJ mol^{-1})

(1) – 1460.50 kJ mol^{-1} (2) – 1350.50 kJ mol^{-1} (3) – 1366.95 kJ mol^{-1} (4) –1361.95 kJ mol^{-1}

Sol: [3] $\Delta_{ng} = 2 - 3 = -1$

$$\Delta H_C = \Delta U + \Delta_{ng} RT$$

$$\Delta H_C = -1364.47 + (-1 \times 8.314 \times 298 \times 10^3)$$

$$\Delta H_C = (-1364.47 - 2.477)\ kT\,mol^{-1}$$

$$\Delta H_C = -1366.95\ KJ\,mol^{-1}$$

90. The most suitable reagent for the conversion of $R - CH_2 - OH \rightarrow R - CHO$ is

(1) CrO_3 (2) PCC (Pyridinium Chlorochromate)

(3) $KMnO_4$ (4) $K_2Cr_2O_7$

Sol: [2] PCC is a mild oxidising agnet

$$R - CH_2OH \xrightarrow{PCC} R - CH_2 - CHO$$

PART A – PHYSICS

1.	(3)	2.	(4)	3.	(1)	4.	(3)	5.	(3)
6.	(2)	7.	(3)	8.	(4)	9.	(2)	10.	(1)
11.	(2)	12.	(1)	13.	(3)	14.	(1)	15.	(1)
16.	(4)	17.	(2)	18.	(1)	19.	(4)	20.	(1)
21.	(1)	22.	(1)	23.	(4)	24.	(1)	25.	(4)
26.	(3)	27.	(1)	28.	(No match)	29.	(2)	30.	(1)

PART B – MATHEMATICS

31.	(1)	32.	(4)	33.	(1)	34.	(4)	35.	(2)
36.	(4)	37.	(4)	38.	(1)	39.	(2)	40.	(2)
41.	(2)	42.	(4)	43.	(3)	44.	(4)	45.	(4)
46.	(3)	47.	(4)	48.	(1)	49.	(4)	50.	(1)
51.	(3)	52.	(2)	53.	(4)	54.	(4)	55.	(3)
56.	(4)	57.	(3)	58.	(1)	59.	(1)	60.	(3)

PART C – CHEMISTRY

61.	(3)	62.	(4)	63.	(4)	64.	(3)	65.	(1)
66.	(3)	67.	(2)	68.	(4)	69.	(4)	70.	(4)
71.	(1)	72.	(4)	73.	(1)	74.	(3)	75.	(2)
76.	(4)	77.	(4)	78.	(2)	79.	(2)	80.	(2)
81.	(3)	82.	(3)	83.	(3)	84.	(1)	85.	(3)
86.	(4)	87.	(3)	88.	(1)	89.	(3)	90.	(2)