A Mathematical Approach To Unsteady Temperature Regulation Of Human Body Due To Arterial Blood Temperature

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Abstract: To understand the physiology of any human body study of temperature regulation of human body has great importance. We drive a Bio-Heat Equation for solving the heat transfer problem in blood vessel in the dermal with environment. We have region assumed that heat transfer in body is combined result of the convection, evaporation and radiation which are in use as boundary conditions for solving any Bio-Heat Equation. Finite difference technique is used in order to find out the temperature distribution of human body due to arterial blood temperature. Normal body temperature is always consider to environment temperature (Tb) at 37oC. Any disorder in the temperature parameter may cause lots of abnormality in the body. In the present paper, we study an unsteady temperature variations of dermal region of human arterial body by changing blood temperature for transient case. The nodal temperatures are noted at different time intervals.

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I. INTRODUCTION

In medical science and biological process ,mathematical technique play fabulous role advancing understand physical to processes. Body temperature plays an important role to regulate every biological Metabolism is main source to system. generate heat continuously within the human body. If this heat is not lost from the body, then the temperature of body will keep on rising continuously. Thus the body heat is continuously lost to the surrounding. The process which maintains thermal balance between the body and surrounding is known as thermal regulation system of the body.

There are many factors affecting body temperature like atmospheric temperature, the body mass and surface area of the body, Human body is composed of three layers namely epi dermis, dermis and subcutaneous. Due to absence of blood vessels in epidermis, the blood circulation in epidermis is negligible. And dermis blood flow is variable while uniform in subcutaneous tissue. The rate of blood flow in SST region is most variable in comparison to other parts of the body.

Many investigations have been made to study thermal responses in normal and

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abnormal conditions. Perl [7] combined differential forms of Fick's perfection principle with heat conduction and matter diffusion equations and metabolic term to obtain equation. He used equation to solve its simple cases by taking all parameters as constant throughout the region. Perl and Hirsch [8] used this equation to test the transient response for measuring local tissue blood flow on dog and rabbit kidney. Trezek and Cooper [6] computed thermal conductivity of tissue by taking all parameters as constant.

Cooper and trezek [2][3] obtain solution of equation in SST region by taking all parameters as constant. Patterson [4][5] made experimental attempts, to determine temperature profiles in skin and subcutaneous region. Saxena [9] solved equation by similarity transformation in SST region. Saxena and Arya [10] used variation finite element methods to solve the problem of steady state temperature distribution in three layered skin and subcutaneous region. Saxena, Arya and Bindra [11] obtained unsteady state temperature distribution in human skin and subcutaneous tissue region, by using the variation finite method and Laplace transform method. Saxena and Gupta[12] discussed the effect of tumor on temperature distribution in human skin. Saxena and Yadav [13] and Shukla [14] types various of temperature solved distribution problem in skin and SST region with thermal injury in steady state case.

Although in this field lot of work has been done by many mathematician, scientists and other researchers but it is not possible to explain the work of everyone .Results obtained by all mentioned mathematicians, scientists and researchers are much under consideration, but their studies are confined to take constant or average values of arterial blood temperature and venous blood temperature is equal to tissue temperature, which are practically not possible in the skin and SST region.

In the present paper we have taken arterial blood temperature as position dependent and venous blood temperature is also taken variable.

II. STATEMENT OF THE PROBLEM

A study of temperature distribution problem in dermal region is the combined study of following processes.

1. Transfer of heat by conduction

2. Transfer of heat by blood circulation.

3. Generation of heat due to metabolic reactions.

4. Heat loss or gain at the skin surface due to convection, radiation and evaporation.

Let Q be the quantity in heat transfer in a tissue element of at time t, then the rate of change of heat transfer Q is denoted

$$\partial Q$$

by ∂t and written as follows:

$$\frac{\partial Q}{\partial t} = \left(\frac{\partial Q}{\partial t}\right)_d + \left(\frac{\partial Q}{\partial t}\right)_p + \left(\frac{\partial Q}{\partial t}\right)_m$$
.....(1)

Where

$$\left(\frac{\partial Q}{\partial t}\right)_d =$$
Rate of change of

heat transfer due to diffusion

$$\left(\frac{\partial Q}{\partial t}\right)_p =$$
Rate of change of

heat transfer due to perfusion

$$\left(\frac{\partial Q}{\partial t}\right)_m =$$
 Rate of change of

heat transfer due to metabolic heat generation.

According to the Fick's law of diffusion $\left(\frac{\partial Q}{\partial t}\right)_d$ can be written as:

$$\left(\frac{\partial Q}{\partial t}\right)_{d} = \rho C \frac{\partial T}{\partial t} \Big/_{d} = div(KgradT)$$
(2)

. . . Where

 ρ = Density of tissue

C = Specific heat of the tissue

K = Thermal Conductivity of the tissue

$$T = \text{Temperature of the tissue}$$

Now $\left(\frac{\partial t}{\partial t} \right)_p$ rate of change of heat transfer due to perfusion is defined with help of Fick's law of perfusion and it can be written as:

$$\left(\frac{\partial Q}{\partial t}\right)_{p} = \rho C \frac{\partial T}{\partial t} \Big/_{p} = \rho_{b} C_{b} \phi_{A} T_{A} - \rho_{b} C_{b} \phi_{V} T_{V}$$
(3)

$$\left(\frac{\partial Q}{\partial t}\right)_{p} = \rho C \frac{\partial T}{\partial t} \Big/_{p} = \rho_{b} C_{b} (\phi_{A} T_{A} - \phi_{V} T_{V})$$
(4)

Where

 ρ_b = Density of blood

 C_b = Specific heat of blood

 ϕ_A = Arterial blood perfusion rate

 T_A = Temperature of arterial blood

 $\phi_V =$ Venous blood perfusion rate

 $T_V =$ Temperature of venous blood

 $\left(\frac{\partial Q}{\partial t}\right)$ ^{*m*} Metabolic heat generation is denoted by S and it can be written as:

$$\left(\frac{\partial Q}{\partial t}\right)_{m} = \rho C \frac{\partial T}{\partial t} \Big/_{m} = S$$
(5)

Now replacing the terms on both side of equation (1) with the help of equation (2), (3) and (4), we get following partial differential equation :

$$\rho C \frac{\partial Q}{\partial t} = Div (KgradT) + \rho_b C_b (\phi_A T_A - \phi_V T_V) + S$$
(6)

In dermal region the difference between ϕ_A and ϕ_V is very-very small, so $\phi_A = \phi_V$. $\rho C \frac{\partial Q}{\partial t} = Div (KgradT) + m_b C_b (T_A - T_V) + S$ (7)

And we have taken

$$T_V = qT$$

At the skin surface heat loss or gain due to convection, radiation and evaporation can be written as,

$$-K\frac{\partial Q}{\partial \eta} = h(T - T_a) + LE$$

(9)

Where

 η = Normal to skin surface

h = Heat transfer coefficient

 $T_a =$ Atmospheric temperature

L = Latent heat of evaporation

E = Rate of sweat evaporation

Here first term on the right hand side of equation (8) shows heat loss and heat gain due to convection and radiation and second term LE determines heat loss due seat evaporation from the skin surface.

It has been discussed earlier that a human body maintains uniform body core temperature which is equal to 370c approximately.

Therefore inner boundary condition can be written as:

$$T = Ta = T_b$$
(10)

Where T_b denoted the body core temperature.

III.MATHEMATICAL ORMULATION

The partial differential equation (6) coupled with equation (8) and equation (7), written in one dimensional unsteady state and compared with "Euler-Lagrange's equation is transformed into the following equivalent form :

$$I = \frac{1}{2} \int_{0}^{a_{3}} \left[K \left(\frac{\partial T}{\partial x} \right)^{2} + mc_{b} (T_{A} - T_{V})^{2} - 2ST + \rho C \frac{\partial T^{2}}{\partial t} \right] dx + \frac{1}{2} h (T_{a} - T)^{2} + LET$$
(11)

We assumed the values T_0, T_1, T_2, T_3 to Tat the points x = 0, $x = a_1$, $x = a_2$ and $x = a_3$ respectively. Here a_1' is the thickness of epidermis and dermis together and a_3' is the total thickness of dermal region. Let T'(x) (r = 1, 2, 3) represents the linear values of T(x) in three sub regions respectively.

SOLUTION

If I_1, I_2 and I_3 are the values of I in three sub-regions then $I = \sum_{i=1}^{3} I_i$.

For solving equation (3.1) we used same conditions which are given in Table 1. The following additional conditions initial and boundary are also used :

$$T(x,0) = 22.87 + px , \quad p > 0$$

$$0 \le x \le a_3$$

$$T(a_3,t) = T_b$$

$$T(a_3,0) = T_b$$

Where p is unknown constant.

We get,

$$\begin{split} I_{1} &= \\ \frac{K_{1}}{a_{1}}(T_{1} - T_{0})^{2} + \frac{h}{2}(T_{0} - T_{a})^{2} + LET_{0} + \frac{1}{6}\rho Ca_{1}\frac{\partial}{\partial t}(T_{0}^{2} + T_{1}^{2} + T_{0}T_{1})^{2} \\ I_{2} &= (T_{2} - T_{1})^{2}A_{1} + (T_{2}a_{1} - T_{1}a_{2})^{2}A_{2} + (T_{2}a_{1} - T_{1}a_{2})(T_{2} - T_{1})A_{3} + (T_{2}a_{1} - T_{1}a_{2})A_{4} \end{split}$$

$$(T_2 - T_1)A_5 + A_6 + \frac{1}{6}\rho C(a_2 - a_1)\frac{\partial}{\partial t}(T_1^2 + T_2^2 + T_1T_2) \quad B_2 = \frac{Mq^2}{2(a_3 - a_2)}$$

$$I_{3} = (T_{3} - T_{2})^{2} B_{1} + (a_{3}T_{2} - a_{2}T_{3})^{2} B_{2} + (T_{3} - T_{2}) B_{3} + (a_{3}T_{2} - a_{2}T_{3}) B_{4}$$
$$(a_{3}T_{2} - a_{2}T_{3}) (T_{3} - T_{2}) B_{5} + B_{6} + \frac{1}{6} \rho C(a_{3} - a_{2}) \frac{\partial}{\partial t} (T_{2}^{2} + T_{3}^{2} + T_{3}^{2}) C(a_{3} - a_{3}) \frac{\partial}{\partial t} (T_{2}^{2} + T_{3}^{2}) C(a_{3} - a_{3}) \frac{\partial}{\partial t} (T_{2}^{2} + T_{3}^{2}) C(a_{3} - a_{3}) \frac{\partial}{\partial t} (T_{3}^{2} - a_{3}) C(a_{3} - a_{3}) \frac{\partial}{\partial t} (T_{3}^{2} - a_{3}) C(a_{3} - a_{3}) \frac{\partial}{\partial t} (T_{3}^{2} - a_{3}) C(a_{3} -$$

 $B_{3} = \frac{-T_{b}qM(2a_{2}^{2} + 3a_{3}^{2} + 2a_{2}a_{3} - 3a_{1}a_{3} - 3a_{1}a_{2})}{6(a_{3} - a_{1})} - \frac{S(a_{3} + a_{2})}{2}$ $B_{4} = \frac{-T_{b}qM}{3} \frac{(a_{3} + a_{2} - 2a_{1})}{(a_{3} - a_{1})} - S$

$$A_{1} = \frac{a_{2}K_{1} - a_{1}K_{3}}{2(a_{2} - a_{1})^{2}} + \frac{(K_{3} - K_{1})(a_{2} + a_{1})}{4(a_{2} - a_{1})} + \frac{q^{2}M(3a_{2}^{2} + 2a_{2}a_{1} + a_{1}^{2})}{24(a_{2} - a_{1})} B_{5} = \frac{Mq^{2}(a_{3} + a_{2})}{2(a_{3} - a_{2})}$$

$$A_2 = Mq^2 \frac{1}{4(a_2 - a_1)},$$

Where,

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$$A_{3} = -\frac{MqT_{b}(a_{2} - a_{1})(a_{1} + 3a_{3})}{12(a_{3} - a_{1})}$$

$$A_4 = -Mq \frac{T_b(a_2 - a_1)}{3(a_3 - a_1)} - \frac{S}{2}$$

$$A_{5} = Mq^{2} \frac{(a_{1} + 2a_{2})}{6(a_{2} - a_{1})},$$
$$A_{6} = M\left(\frac{T_{b}}{a_{3} - a_{1}}\right)^{2} \frac{(a_{2} - a_{1})^{3}}{8}$$

$$B_1 = \frac{q^2 M}{6} \frac{\left(a_3^2 = a_2^2 + a_3 a_2\right)}{\left(a_3 - a_2\right)} = \frac{K_3}{2(a_3 - a_2)}$$

 $B_{6} = M \left(\frac{T_{b}}{a_{3} - a_{2}}\right)^{2} \frac{\left(a_{3}^{2} + a_{2}^{2} + a_{3}a_{2} - 3a_{1}a_{2} - 3a_{1}a_{3} + 3a_{1}^{2}\right)\left(a_{3} - a_{2}\right)}{6}$

Since, T_3 is equal to body core temperature, so we minimize I for T_0, T_1, T_2, T_3 . Accordingly, we get following system of algebraic equations :

$$L_{1}T_{0} + L_{2}T_{1} + \frac{1}{6}\rho C \left[2a_{1}\frac{\partial T_{1}}{\partial t} + a_{1}\frac{\partial T_{2}}{\partial t} \right]$$

$$= W_{0} \qquad (12)$$

$$M_{1}T_{0} + M_{2}T_{1} + M_{3}T_{2} + \frac{1}{6}\rho C \left[a_{1}\frac{\partial T_{0}}{\partial t} + 2a_{2}\frac{\partial T_{1}}{\partial t} + (a_{2} - a_{1})\frac{\partial T_{2}}{\partial t} \right] = W_{1}$$

$$(13)$$

$$N_{1}T_{1} + N_{2}T_{2} + \frac{1}{6}\rho C \left[(a_{2} - a_{1})\frac{\partial T_{1}}{\partial t} + 2(a_{3} - a_{2})\frac{\partial T_{2}}{\partial t} \right] = W_{2}$$

$$(14)$$

Where,

$$L_{1} = \frac{K_{1}}{a_{1}} + h$$
$$L_{2} = -\frac{K_{1}}{a_{1}}$$

$$W_0 = hT_a - LE$$
$$M_1 = -\frac{K_1}{a_1}$$

$$M_{2} = \frac{K_{1}}{a_{1}} + 2A_{1} + 2a_{2}^{2}A_{2} - 2a_{2}A_{5}$$
$$W_{1} = A_{3} - a_{2}A_{4}$$

$$N_1 = -2A_1 - 2a_1a_2A_2 + (a_1 + a_2)A_5$$

$$N_2 = 2A_1 + 2a_1^2A_2 - 2a_1A_5 + 2B_1 + 2a_3^2B_2 - 2a_3B_5$$

$$W_2 = [2B_1 + 2a_2a_3B_2 - (a_2 + a_3)B_5]T_3 - A_3 + a_1A4 + B_3 - a_3B_4$$

Equation (11), (12) and (13) are written in the following matrix form :

$$C\dot{T} = -\overline{K}T + W$$
(15)

Where

$$C = \frac{1}{6} \rho c \begin{bmatrix} 2a_1 & a_1 & 0\\ a_1 & 2a_2 & a_{2-}a_1\\ 0 & a_2 - a_1 & 2(a_3 - a_1) \end{bmatrix}$$

$$\dot{T} = \begin{bmatrix} \frac{\partial T_0}{\partial t} \\ \frac{\partial T_1}{\partial t} \\ \frac{\partial T_2}{\partial t} \end{bmatrix}$$
$$\overline{K} = \begin{bmatrix} L_1 & L_2 & 0 \\ M_1 & M_2 & M_3 \\ 0 & N_1 & N_2 \end{bmatrix}$$
$$T = \begin{bmatrix} T_0 \\ T_1 \\ T_2 \end{bmatrix}_{\&} W = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}$$

,

This system of equation (14) is solved by Crank-Nilcoson method, therefore we get

$$\left(C + \frac{1}{2}\overline{K}\Delta t\right)T^{V+1} = \left(C - \frac{1}{2}\overline{K}\Delta t\right)T^{V} + W\Delta t$$
(16)

Where Δt is time interval and V= 0, 1, 2,n (number of interval with respect to time).

VII. NUMERICAL RESULT AND DISCUSSION

For numerical result we make use of following values of physical quantities.

$$h = 0.15 \times 10^{-3} \text{ cal/cm2} 5^{\circ} \text{ C}$$
$$L = 579 \text{ cal/gm}$$
$$K_{1} = 0.5 \times 10^{-3} \text{ cal/cm} 5^{\circ} \text{ C}$$
$$K_{3} = 1.0 \times 10^{-3} \text{ cal/cm} 5^{\circ} \text{ C}$$

 $T_{a} = 33^{\circ} C$ $T_{b} = 37^{\circ} C$ $a_{1} = 0.10cm$ $a_{2} = 0.35cm$ $a_{3} = 0.50cm$ $M = 0.52 \times 10^{-3} \text{ cal/cm } 5^{\circ}$ $S = 0.3 \times 10^{-3} \text{ cal/cm } 3 \text{ S}$ $E = 0.008 \times 10^{-3} \text{ gm/ } \text{ cm}^{2} S$ $\rho = 1.05 \text{ g/ } \text{ cm}^{3}$ $C = 0.83 \text{ cal } \text{/ gm}^{\circ} c$

Using, these numerical values, we get the following values of nodal temperature at different time.

Table 1: Temperature distribution of dermis epi-dermis and subcutaneous with respect to time

Time (t)	-	_	_	Time (f)	_	_	_
(Min)	T_0	T_1	T_2	(Min)	T_0	T_1	T_2
0.00	22.87	25.70	32.76	3.10	32.08	33.11	35.57
0.05	23.75	25.78	32.88	3.15	32.12	33.15	35.58
0.10	24.26	26.01	32.94	3.20	32.16	33.18	35.60
0.15	24.65	2627	32.09	3.25	32.20	33.21	35.61
0.20	25.22	26.69	33.13	3.30	32.23	33.24	35.62
0.25	25.48	26.96	33.19	3.35	32.26	33.27	35.63
0.30	25.74	27.21	33.26	3.40	32.29	33.30	35.64
0.35	26.00	27.44	33.54	3.45	32.32	33.33	35.65
0.40	2625	27.67	33.42	3.50	32.35	33.36	35.66
0.45	26.49	27.89	33.50	3.55	32.38	33.39	35.67
0.50	26.72	28.18	33.58	3.60	32.41	33.41	35.68
0.55	26.92	2830	33.65	3.65	32.44	33.44	35.69
0.60	27.15	28.50	33.72	3.70	32.47	33.46	35.70
0.65	2735	28.69	33.79	3.75	32.49	33.48	35.71
0.70	27.55	28.87	33.86	3.80	32.51	33.50	35.72
0.75	27.74	29.04	33.93	3.85	32.53	33.52	35.73
0.80	27.92	29.21	34.00	3.90	32.55	33.54	35.74
0.85	28.09	2937	34.07	3.95	32.57	33.56	35.75
0.90	28.26	29.53	34.13	4.00	32.59	33.58	35.76
0.95	28.44	29.66	34.19	4.05	32.61	33.60	35.77
1.00	28.58	29.83	34.25	4.10	32.63	33.62	35.78
1.05	28.73	29.97	3431	4.15	32.65	33.64	35.79
1.10	28.88	30.11	3436	4.20	32.67	33.66	35.80
1.15	29.02	30.24	34.41	4.25	32.69	33.68	35.80
1.20	29.16	30.37	34.46	4.30	32.71	33.70	35.81
1.25	29.29	30.50	34.51	4.35	32.73	33.72	35.82
1.30	29.42	30.62	34.56	4.40	32.75	33.74	35.83
1.35	29.55	30.74	34.61	4.45	32.77	33.75	35.83
1.40	29.67	30.85	34.66	4.50	32.79	33.77	35.84

1.45	29.79	30.96	34.70	4.55	32.81	33.78	35.84
1.50	29.90	31.07	34.74	4.60	32.82	33.80	35.84
1.55	30.01	31.17	34.78	4.65	32.84	33.81	35.85
1.60	30.12	3127	34.82	4.70	32.85	33.82	35.86
1.65	30.22	3136	34.86	4.75	32.86	33.83	35.86
1.70	30.32	31.45	34.90	4.80	32.87	33.84	35.86
1.75	30.41	31.54	34.94	4.85	32.88	33.85	35.87
1.80	30.50	31.63	34.97	4.90	32.89	33.86	35.87
1.85	30.59	31.71	35.00	4.95	32.90	33.87	35.87
1.90	30.68	31.79	35.03	5.00	32.91	33.88	35.88
1.95	30.76	31.87	35.06	5.05	32.92	33.89	35.88
2.00	30.84	31.94	35.09	5.10	32.93	33.90	35.88
2.05	30.94	32.01	35.12	5.15	32.94	33.91	35.89
2.10	30.99	32.08	35.15	5.20	32.95	33.92	35.89
2.15	31.06	32.15	35.18	5.25	32.96	33.93	35.90
2.20	31.13	32.22	35.21	5.30	32.97	33.94	35.90
2.25	31.20	32.28	35.24	5.35	32.98	33.95	35.90
2.30	31.27	32.34	35.26	5.40	32.99	33.96	35.91
2.35	3133	32.40	35.28	5.45	33.00	33.97	35.91
2.40	3139	32.46	3530	5.50	33.01	33.98	35.91
2.45	31.45	32.52	35.32	5.55	33.02	33.99	35.92
2.50	31.51	32.57	35.35	5.60	33.03	34.00	35.92
2.55	31.57	32.62	3537	5.65	33.04	34.00	35.92
2.60	31.62	32.67	3539	5.70	33.05	34.01	35.92
2.65	31.67	32.72	35.41	5.75	33.06	34.02	35.93
2.70	31.77	32.77	35.43	5.80	33.07	34.03	35.93
2.75	31.82	32.82	35.45	5.85	33.08	34.03	35.93
2.80	31.82	32.86	35.47	5.90	33.08	34.04	35.93
2.85	3191	32.90	35.49	5.95	33.09	34.04	35.94
2.90	31.96	32.95	35.51	6.00	33.09	34.05	35.94
2.95	32.00	32.99	35.53	6.05	33.10	34.05	35.94
3.00	32.04	33.03	35.54	6.10	33.10	34.06	35.94
3.05	32.08	33.07	35.55	6.15	33.11	34.06	35.94

In present paper mathematical model has been developed to analyze the temperature variation in dermal region. The total thickness is taken 0.95cm.The thickness of subcutaneous; dermis and epidermis are 0.50cm, 0.35cm and 0.10cm respectively.

Graphs are plotted between temperature T and distance x for different values of q and different atmospheric temperature. Numerical solutions are obtained for sub dermis, dermis and epidermis region.

Initially it is assumed that SST region is fully insulated. So temperature of each layer at time t=0 is equal to the 37° C.

In the entire graph, heat loss from epidermis layer is more than that of the dermis and SST region due to evaporation. Temperature variation is seen by changing the value of q. It is observed that the fall in tissue temperature is more at same rate of evaporation and lower atmospheric temperature. Also the tissue temperature decreases with the rise in venous blood temperature.



Fig. 1 Temperature distribution in dermal region with respect to Time

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