Engineering and Technology Journal for Research and Innovation (ETJRI)
ISSN 2581-8678, Volume IV, Issue II, Jun 2022

Ergodic Sum-Rate Analysis of two user- NOMA System over Generalized Fading Channels

Priyank Sharma
Assistant Professor, Amity University Madhya Pradesh, Gwalior, ASET-ECE, India,
priyank.sharma35@gmail.com

***Abstract* – The ergodic sum-rate of downlink non-orthogonal multiple access (NOMA) system with static power allocation is investigated in this paper over $\kappa - \mu$ and $\eta - \mu$ fading environment. Further, the analytical expression of the average sum-rate is derived using the probability density function (PDF) approach. In addition, the impact of fading parameters and the rate of users on the ergodic sum-rate are examined, and the correctness of analytical results is validated through Monte Carlo (MC) simulation.**

Keywords – Generalized fading channels, Non-Orthogonal Multiple Access (NOMA), Sum-rate.

1. INTRODUCTION

The need for spectral efficiency, user fairness, and massive connectivity is of soul interest in 5G and beyond the wireless system [1]. In par with this, non-orthogonal multiple access (NOMA) has been identified as a promising multiple access technique [2], and many works have been reported in [3] – [8] that analyse the sum-rate, and the outage of the NOMA system over various fading channels. To be precise, in [3] multiuser beam-forming technique is proposed for the NOMA scheme over Rayleigh distributed environment, and simulation results show that the proposed scheme shows a better sum-rate. In [4], a novel power allocation technique is proposed for the NOMA network under an independent complex Gaussian channel. The simulation results show that the proposed technique improves the capacity when compared with the orthogonal multiple access (OMA) scheme. The outage performance of NOMA is analyzed over Rician fading channel in [5]. The simulation results show that Rician fading channel achieves a better outage performance compared to Rayleigh distributed environment. In [6], the downlink NOMA system is analyzed with static power allocation over Nakagami-m fading channel in terms of an outage. In [7], the outage and ergodic sum-rate of downlink NOMA are analyzed over Rayleigh fading channel with randomly deployed users, and simulation results show that the proposed scheme outperforms the OMA scheme. The NOMA network is analyzed over $\eta - \mu$ fading channel in terms of outage in [8]. The $\eta - \mu$ and $\kappa - \mu$ distributions are known to be more generalized models for practical fading environments in [9] due to their capability to model various fading environments by varying the parameter values.

So far, the downlink NOMA network is analyzed over Rayleigh, Rician, Nakagami-m, $\boldsymbol{\kappa - \mu}$, and $\eta - \mu$ fading distribution in terms of the outage. In this paper, the ergodic sum rate of the downlink NOMA system is analyzed over $\eta - \mu$ and $\boldsymbol{\kappa - \mu}$ fading distribution.

To the best of the author's knowledge, the analysis of ergodic sum-rate in downlink NOMA system over $\boldsymbol{\kappa - \mu}$ and $\eta - \mu$ distribution with the PDF method has not been reported in the literature. The major contributions of this paper are listed below.

1) A pair of users i.e., near and far users are chosen for analysis. The ergodic sum-rate expressions are derived over $\eta - \mu$ and $\boldsymbol{\kappa - \mu}$ fading channels. The analytical results are compared with the MC simulation.
2) The average sum-rate and rate of users analysis is carried out by varying η and $\boldsymbol{\kappa}$ values.

The remaining part of the paper is organized as follows. Section 2 describes the system model with its signal-to-interference-noise-ratio (SINR) equations. The

Engineering and Technology Journal for Research and Innovation (ETJRI)
ISSN 2581-8678, Volume IV, Issue II, Jun 2022

mathematical expressions of the average sum-rate for the considered system model are derived in Section 3. Simulation results with discussion are presented in Section 4. The conclusion and future scope are provided in Section 5

2. SYSTEM MODEL

Consider a downlink NOMA system in which base station (BS) communicates the information to users as illustrated in Fig.1, wherein the channel gains of the two users are arranged as $|h_1|^2 \geq |h_2|^2$. From the NOMA principle, the power allocation factor for users are arranged as $\alpha_1^2 \leq \alpha_2^2$. Further, it is assumed that BS has the knowledge of the channel state information (CSI) of users.

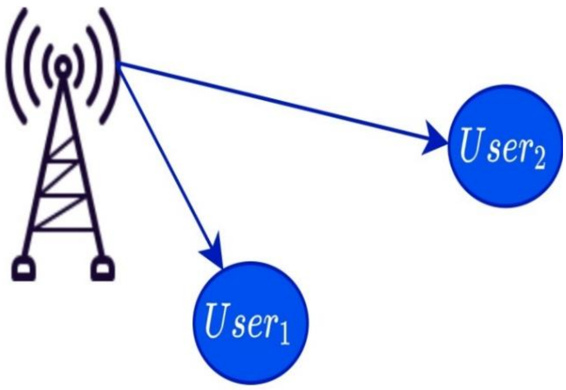

Fig. 1. System Model for downlink NOMA network.

The message signal of both the users is superimposed at the BS with a different power level and transmitted to the destination (D). It is mathematically expressed as

$$x_{BS} = \sum_{m=1}^{2} \alpha_m w_m, \quad (1)$$

where w_m is the message signal intended for m^{th} user and α_m is the corresponding power allocation coefficient. Therefore, the received signal at D is expressed as,

$$y_m = \sqrt{P_s} h_m \, x_{BS} + n_m \,, \quad (2)$$

where h_m is m^{th} user channel coefficient from BS to destination, P_s is the transmit power by BS, and $n_m \sim (0, \sigma^2)$ is the additive white Gaussian noise (AWGN) at m^{th} user. Thus, the SINR of user2 at D is mathematically expressed as,

$$SINR_2 = \frac{|h_2|^2 \alpha_2^2 \rho}{|h_1|^2 \alpha_1^2 \rho + 1}, \quad (3)$$

where $\rho = \frac{P_s}{\sigma^2}$ represents the transmit SNR and $|h_2|^2$ is the channel gain of user2. If SIC is perfect, the received SINR at user1 for decoding its own signal at the destination [10] as,

$$SINR_1 = |h_1|^2 \alpha_1^2 \rho, \quad (4)$$

The channels between BS and users are assumed to be independent and identically distributed (i.i.d.). In this paper, two fading scenarios are considered to analyze the system under consideration. In case 1, the channel coefficients are $\eta - \mu$ distributed, and in case 2, the channel coefficients are $\boldsymbol{\kappa - \mu}$ distributed. In the next section, Ergodic Sum-Rate for the considered system is been analyzed.

4. ERGODIC SUM-RATE ANALYSIS

In this section, closed-form expressions of the ergodic sum-rate are derived over a generalized fading environment for the considered system model. Mathematically, the sum rate is defined as

$$C = log_2(1 + SINR_1) + log_2(1 + SINR_2) \quad (5)$$

From (3) and (4), (5) can be written as

$$C = log_2(1 + x_1 \alpha_1^2 \rho) + log_2\left(\frac{1 + x_2 \rho}{x_2 \alpha_1^2 \rho + 1}\right) \quad (6)$$

where $x_m = |h_m|^2$. The ergodic sum rate can be expressed as

$$\langle C \rangle = \langle log_2(1 + x_1 \alpha_1^2 \rho) \rangle + \langle log_2(1 + x_2 \rho) \rangle - \langle log_2(x_2 \alpha_1^2 \rho + 1) \rangle \quad (7)$$

where $\langle . \rangle$ represent the ensemble average which can be written as in (8)

$$\langle log_2(1 + C x_m) \rangle = \int_0^\infty log_2\,(1 + Ct) f_{x_m}(t) \quad (8)$$

Engineering and Technology Journal for Research and Innovation (ETJRI)
ISSN 2581-8678, Volume IV, Issue II, Jun 2022

where the values of C are calculated depending upon (7) and $f_{x_m}(x)$ is the probability density function (PDF) of x_m, and written as [11]

$$f_{x_1}(x) = 2[F_{\tilde{x}}(x)]f_{\tilde{x}}(x)\,, f_{x_2}(x) = 2[1 - F_{\tilde{x}}(x)]f_{\tilde{x}}(x) \quad (9)$$

where $\tilde{x}$ is the unordered variable and $F_{\tilde{x}}(x)$ is the cumulative distribution function (CDF) of $\tilde{x}$. From (7) and (8)

$$E[log_2(1 + Cx_1)] = 2\int_0^{\infty} log_2\,(1 + Cx)\,[F_{\tilde{x}}(x)]f_{\tilde{x}}(x)dx \quad (10)$$

$$E[log_2(1 + Cx_2)] = 2\int_0^{\infty} log_2\,(1 + Cx)[1 - F_{\tilde{x}}(x)]f_{\tilde{x}}(x)dx \quad (11)$$

4.1 Ergodic Sum-Rate for $\eta - \mu$ Fading Channel:

If h_m follows the $\eta - \mu$ distribution, then the PDF of $\tilde{x}$ is given as [9]

$$f_{\tilde{x}}(t) = \frac{2\sqrt{\pi}\,\mu^{\mu+0.5}\,h^{\mu}}{\Gamma(\mu)H^{\mu-0.5}\bar{t}^{\mu+0.5}}\, e^{\frac{-2\mu h t}{t}}\, t^{\mu-0.5}\, I_{\mu-0.5} \quad (13)$$

The CDF of $\tilde{x}$ is $Y_{\mu}\left(\frac{H}{h}, \sqrt{\frac{2h\mu t}{\bar{t}}}\right)$, where $Y_{\mu}(x,y)$ is Yacoub's integral [9]. $F_{\tilde{x}}(x)$ is further simplified as

$$F_{\tilde{x}}(t) = 1 - \frac{\Gamma\left(2\mu\ \frac{2h\mu t}{\bar{t}}\right)}{\Gamma(2\mu)} \quad for\ \eta \to 1 \quad (14)$$

and

$$F_{\tilde{x}}(t) = 1 - \frac{\Gamma\left(\mu, \left[1 - \frac{H}{h}\right]\ \frac{2h\mu t}{\bar{t}}\right)}{\Gamma(\mu)} \quad for\ \eta \to 0 \quad (15)$$

where $\Gamma(a,z) = \int_z^{\infty} e^{-t}\, t^{a-1}dt$ is the incomplete gamma function [12]. Substituting $f_{\tilde{x}}(t)$ and $F_{\tilde{x}}(t)$ in (7), the average sum-rate of the considered system model is numerically evaluated

4.2 Ergodic Sum-Rate evaluated for $\kappa - \mu$ fading Channel:

If h_m follows the $\kappa - \mu$ distribution, the PDF of $\tilde{x}$ is given as [9]

$$f_{\tilde{x}}(t) = \frac{\mu\,(1+k)^{\frac{\mu+1}{2}}}{\bar{t}^{\frac{\mu+1}{2}} e^{\mu k} k^{\frac{\mu-1}{2}}}\, e^{\frac{-\mu(1+\kappa)t}{t}}\, t^{\frac{\mu-1}{2}}\, I_{\mu-1}\left(2\mu\sqrt{\frac{k(}{}}\right. \quad (16)$$

The CDF of $\tilde{x}$ is $F_{\tilde{x}}(t) = 1 -$ *Qμ2κμ,2(k+1)μtt*, where *Qμx,y* is the generalized Marcum Q-function [9]. Substituting the values of $f_{\tilde{x}}(x)$ and $F_{\tilde{x}}(x)$ into (7), the ergodic sum-rate of the considered system model is numerically evaluated.

5. RESULTS AND DISCUSSION

The derived analytical expressions and MC simulation of considered system model are validated in MATLAB R2021a with 10^5 iterations. In our system model, two users are considered i.e., user1 and user2 with the power allocation factors $(\alpha_1^2, \alpha_2^2) = (0.75, 0.25)$ for user1 and user2, respectively.

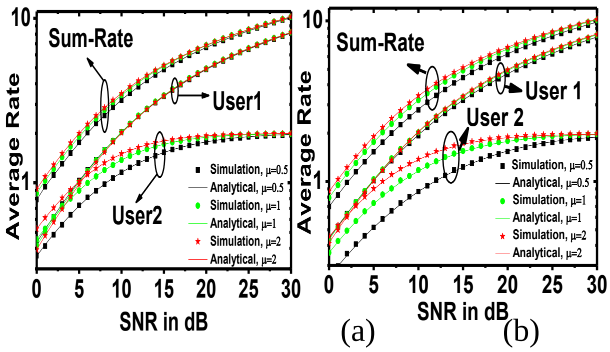

Fig. 2. Average Rate with $\eta - \mu$ fading distribution at (a) η → 1 and (b)η → 0.

The average sum-rate of the system over $\eta - \mu$ fading distribution is depicted in Fig. 1 at different fading values. It is observed that as transmit SNR increases, the sum-rate increases for both $\eta \to 1$ and $\eta \to 0$ value. Further, it is observed that as

ISSN 2581-8678, Volume IV, Issue II, Jun 2022

μ value is increases, the number of clusters also increases which leads less severity of fading. This in turn, the sum-rate gets increases at lower SNR range. Furthermore, the impact of another fading parameter i.e., η can be observed from Fig. 2 (a) and 2(b), respectively. As η values increases, the sum-rate and rate of both users is increased. Finally, it is inferred that user2 shows a less average rate when compared with user1. This is due to the fact that user2 is assigned with less channel gain and more power.

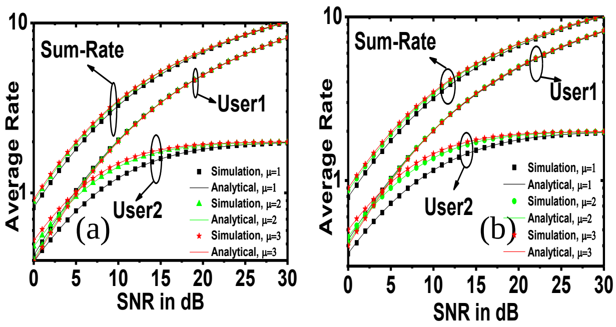

Fig. 3. Average Rate with $\kappa - \mu$ fading distribution at (a) $\kappa \rightarrow 1$ and (b) $\kappa \rightarrow 0$.

The sum-rate and rate of each users in the considered system model is shown in Fig.3 with $\kappa - \mu$ fading distribution at different fading values μ=1, 2, 3; and κ=0 and κ=1. It is observed from the figure that, as SNR increases, the rate of users and sum-rate are also increasing. Further, it is observed that user2 increases with SNR and then becomes saturated at a higher value of SNR. The impact of fading parameters can be concluded that as the value of fading parameters increases, the sum-rate and each user rate also gets increased.

6. CONCLUSION

In this paper, the ergodic sum-rate for the downlink NOMA system over $\eta - \mu$ and $\kappa - \mu$ fading distribution is analyzed. The analytical expressions of ergodic sum-rate is derived and validated with MC simulation. Further, it is observed that as SNR increases, sum-rate and rate of the user1 also increases. However, at high SNR, the average rate of user2 becomes saturated. Finally it is concluded that if fading parameters are increased, the sum-rate and rate of individual users increases.

REFERENCES

[1]. Ding, Zhiguo, Yuanwei Liu, Jinho Choi, Qi Sun, Maged Elkashlan, I. Chih-Lin, and H. Vincent Poor. "Application of non-orthogonal multiple access in LTE and 5G networks." IEEE Communications Magazine 55, no. 2 (2017): 185-191.

[2]. Ding, Zhiguo, Xianfu Lei, George K. Karagiannidis, Robert Schober, Jinhong Yuan, and Vijay K. Bhargava. "A survey on non-orthogonal multiple access for 5G networks: Research challenges and future trends." *IEEE Journal on Selected Areas in Communications* 35, no. 10 (2017): 2181-2195.

[3]. Kim, B., Lim, S., Kim, H., *et al.*: 'Non-orthogonal multiple access in a downlink multiuser beamforming system' Proc. IEEE MILCOM, USA, 2013, pp. 1278-1283.

[4]. Wang, C.L., Chen, J.Y., and Chen, Y.J.: 'Power Allocation for a Downlink Non-Orthogonal Multiple Access System', *IEEE Wirel. Commun. Lett.*, 2016, 5, (5), pp. 532–535.

[5]. K. Srinivasarao and M. Surendar, "Outage Analysis of Downlink Non-Orthogonal Multiple Access Scheme Over Rician Fading Channel," 2020 IEEE 4th Conference on Information & Communication Technology (CICT), 2020, pp. 1-5, doi: 10.1109/CICT51604.2020.9312114.

[6]. Hou, Tianwei, Xin Sun, and Zhengyu Song. "Outage performance for non-orthogonal multiple access with fixed power allocation over Nakagami-${m} $ fading channels." *IEEE Communications Letters* 22, no. 4 (2018): 744-747.

[7]. Ding, Z., Yang, Z., Fan, P. and Poor, H.V., 2014. On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users. *IEEE signal processing letters*, *21*(12), pp.1501-1505.

[8]. Morales-Jiménez, David, and Jose F. Paris. "Outage probability analysis for η-μ fading channels." IEEE Communications Letters 14, no. 6 (2010): 521-523.

Engineering and Technology Journal for Research and Innovation (ETJRI)
ISSN 2581-8678, Volume IV, Issue II, Jun 2022

[9]. Yacoub, M.D.: 'The κ-μ distribution and the η-μ distribution', *IEEE Antennas Propag. Mag.*, 2007, *49*, (1), pp. 68-81.
[10]. Benjebbour, A., Saito, Y., Kishiyama, Y., *et al.*: 'Concept and practical considerations of non-orthogonal multiple access (NOMA) for future radio access', Proc. IEEE ISPACS, Japan, 2013, pp. 770–774.
[11]. David, H.A., Nagaraja, H.N.: 'Order Statistics' (Wiley, USA, 2003, third edition).
[12]. Ryzhik, I.M. and Gradshteyn, I.S.: 'Table of integrals, series, and products' (Elsevier, USA, 2007, seventh edition).